Decision-Oriented Rough Set Methods

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Abstract. Rough set theory is a very effective multi-attribute decision analysis tool. The paper reviews four decision-oriented rough set models and methods: dominance-based rough set, three-way decisions, multigranulation decision-theoretic rough set and rough set based multi-attribute group decision-making model. We also introduce some of our group's works under these four models. Several future research directions of decision-oriented rough sets are presented in the end of the paper.

Keywords: Rough set · Multi-attribute decision making · Group decision making

1 Introduction

Multi-attribute decision making involves the examination of a discrete set of alternatives that are described along with several attributes. In the past several decades, much attention has been paid to multi-attribute decision making from an expert to a group of experts, determined decision environment to uncertain decision environment, and single method to integrated method. There are mainly four kinds of tasks for decision makers to solve multi-attribute decision-making problems [1]:

- Choice: to identify the best object or select a limited set of the best objects from the set of alternatives.
- Ranking: to construct a rank-ordering of the alternatives from the best to the worst ones.
- Sorting: to classify/sort the alternatives into predefined homogenous groups.
- Description: to identify the major distinguishing attributes of the alternatives, explore the relations of attributes, and build decision rules for illustrating how to make a decision.

Until now, a variety of methods have been suggested to solve multi-attribute decision-making problems. The representative ones are Multi-attribute Utility Theory (MAUT) [2], Analytic Hierarchy Process (AHP) [3], ELimination Et choix Traduisant la REalité (ELECTRE) [4], Preference Ranking Organization

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Method for Enrichment Evaluations (PROMETHEE) [5], Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) [6], Rough Sets (RS) [7,8], et al..

In practice, decision makers are facing with the following three decision scenarios [8]: selecting the most importance attributes for making the right decisions, giving a prescription on how to make a decision under specific circumstances, explaining a decision in terms of circumstances under which the decision has been made.

Rough set theory proposed by Pawlak in [7,8] is one of the most important methods for solving multi-attribute decision-making problems in the three scenarios mentioned above. It has demonstrated its superior performances via attribute reduction and rule acquisition in multi-attribute decision making [7–11].

Attribute reduction offers a systematic framework for selecting the independent and essential attributes in a decision system. In the past three decades, many reduction techniques have been developed for selecting the important or essential attributes from different aspects. The discernibility matrix reduction [11] and heuristic reduction [12–15] provide effective tools for selecting the suitable attribute set from the aspects of algebra and information. Rule acquisition is another important contribution in decision-oriented rough set methods. Decision rules derived from the decision examples provide an easy-to-comprehend way for illustrating the decision foundation of decision makers. Researchers have designed many rule induction algorithms in different rough set models. LEM2 is one of the most widely used rule induction algorithms [16]. DOMLEM in [17] extracts dominance rules from decision systems under the framework of dominance-based rough set. In 2001, Grzymala [18] proposed a new rule induction algorithm called MODLEM, in which the discretization process and rule induction are performed at the same time. Several variations of these algorithms have been proposed in many other extended rough set models [14,15,19].

For the remarkable performances in selecting important attributes and explaining decisions, rough set has become an important multi-attribute decision model, and the rough decision methods have received special favors of researchers and practitioners from different fields. The paper introduces four decision-closely-related rough set models and some of our group's works under the four models. We also discuss several research directions of the decision-oriented rough set methods.

2 Decision-Oriented Rough Set Models and Methods

Rough set theory is an effective tool for multi-attribute decision analysis [7]. Over the past thirty years, researchers have proposed different extensions of rough set such as fuzzy rough set [20–22], dominance-based rough set [10,17,23–26], probabilistic rough set [27–31], covering-based rough set [32], multigranulation rough set [33–36], and others. All of these models can be applied to aid the multi-attribute decision making for different application demands. This section aims to introduce the following four decision-closely-related rough set models.

(1) Dominance-based rough set model

In multi-attribute/criteria decision-making problems, there in nature exist preference structures between condition attributes and decision attributes. Greco et al. proposed the eminent extended rough set model called dominance-based rough set (DRS) model by replacing the original indiscernibility relation with the dominance relation, which takes into consideration of the **dominance principle** [10,25,26], *if the performances of an object are all not worse than the other on all considered attributes, its assignment should not be made to a worse decision class than the others.*

Dominance-based rough set considers an ordered decision information system $S = (U, C \cup d, V, f)$, where d ($f(x, d)$ is single-valued) is the overall preference called the decision and all the elements of C are criteria (attributes with preference structures). Furthermore, $\mathbf{CL} = \{CL_1, CL_2, ..., CL_r\}$ is a set of classes of U, such that an alternative x belongs to one and only one class $CL_s \in \mathbf{CL}$ and each element of CL_r is preferred (strictly or weakly) to each element of CL_s if $s > r$. The dominance relation $xD_Py(P \subseteq U)$ means that x dominates y with respect to each attribute/criterion $p \in P$. Given $P \subseteq C$ and $x \in U$, the dominating and dominated sets of x with respected to P are defined as

$$D_P^+(x) = \{y \in U : yD_Px\} \tag{1}$$

and

$$D_P^-(x) = \{y \in U : xD_Py\}. \tag{2}$$

The upward and downward unions of dominance decision classes are defined as

$$CL_t^{\geq} = \bigcup_{s \geq t} CL_s, CL_t^{\leq} = \bigcup_{s \leq t} CL_s. \tag{3}$$

The P- lower and upper approximations of $CL_t^{\geq}$ are

$$\underline{P}(CL_t^{\geq}) = \{x \in U : D_P^+(x) \subseteq CL_t^{\geq}\} \tag{4}$$

and

$$\overline{P}(CL_t^{\geq}) = \bigcup_{x \in CL_t^{\geq}} D_P^+(x), \tag{5}$$

respectively.

The P- lower and upper approximations of $CL_t^{\leq}$ are defined as

$$\underline{P}(CL_t^{\leq}) = \{x \in U : D_P^-(x) \subseteq CL_t^{\leq}\} \tag{6}$$

and

$$\overline{P}(CL_t^{\leq}) = \bigcup_{x \in CL_t^{\leq}} D_P^-(x), \tag{7}$$

respectively.

For accomplishing the sorting and description tasks, DRS permits to construct the approximations of dominance class set on basis of the dominance preference knowledge bases and then to produce the corresponding decision rules satisfying the dominance principle [10,24,26]. It has also been proved that the preference model in the form of dominance rules is more general than the MAUT or the outranking models, and is more understandable for users because of its natural syntax. Researchers have studied the dominance-based rough sets in various of complex decision sceneries [37,38]. Moreover, the decision rules derived by DRS can also be used to give a recommendation for the choice or ranking problem [23,39].

Recently, Song and Liang [40] proposed a two-grade DRS-based approach for multi-attribute decision making to obtain a complete rank of alternatives, which are described with interval attributes taking values in the form as $[a^L(x_i), a^U(x_i)]$.

In the first grade, $DDI_a(x_i, x_j)$ and $DDI_A(x_i, x_j)$ are defined as directional distance indexes for the ordered pair (x_i, x_j) under an attribute a and an attribute set A. Then alternatives can be ranked according to the entire directional distance index $DDI_A(x_i)$. It is deserved to point out that there often exit some objects being put into the same place.

In the second grade, a new order mutual information $E(A^{\geq}; B^{\geq})$ is defined to depict the consistence degree between two dominance ordered structures. Accordingly, the importance of attributes is depicted via the order mutual information. Finally, a complete rank is obtained by the whole weighted directed distance indexes $DDI_A^*(x_i, x_j)$.

The DRS-based two-grade multi-attribute decision approach establishes the complete rank for alternatives by analyzing the dominance structures among criteria with interval data. The approach has been successfully applied to solving stock selection problems in China and also shown the effectiveness in risk-averse multi-attribute decision making.

(2) Three-way decision theory

In practical decision making, adequate judgment information brings an immediately “acceptance” or “rejection” decision, and the not-sufficient judgment information usually causes a “deferment” decision. This kind of decision making extends the two-way decision. It is used in everyone’s daily life and widely applied in many social works as medical decision making, social judgement theory, hypothesis test in statistics, management sciences, and peering review process. Yao first gave the unified formal description of three-way decisions in [41]. The two-state three-way decision model with an evaluation function is introduced in what follows. Detailed description of the other three-way decision models can be found in [41].

An evaluation function $v : U \to L$ is used to estimate the states of objects in the universe U, where $(L, \preceq)$ is a totally ordered set. By introducing a pair of threshold (α, β), ($\beta \prec \alpha$ i.e. $\beta \prec \alpha$ and $\beta \neq \alpha$) and the evaluation v, the three regions of three-way decisions are constructed as

$$POS_{(\alpha,\beta)}(v) = \{x \in U | v(x) \succeq \alpha\}, \tag{8}$$

$$NEG_{(\alpha,\beta)}(v) = \{x \in U | v(x) \preceq \beta\} \tag{9}$$

and

$$BND_{(\alpha,\beta)} = \{x \in U | \beta \prec v(x) \prec \alpha\}. \tag{10}$$

Decision-theoretic rough set (DTRS) [42] is a well-formed concrete three-way decision model, in which evaluation function v is interpreted as the possibility of an object belonging to the objective region, L is specialized as the unit interval [0,1] and α, β are the minimum cost thresholds. DTRS gives the thresholds α, β and evaluation function $v(x)$ the clear semantic interpretations by minimizing the decision costs with Bayesian theory [30]. The superiority of DTRS has invoked the interest of many scholars, and then many studies about DTRS have been done in recent years [43–45]. Greco et al. [43] combined the decision-theoretic rough set with the dominance rough set. Yao and Herbert [44] combined DTRS with game-theoretic rough set. Liang [45] proposed a naive model of intuitionistic fuzzy decision-theoretic rough sets and applied it to single-period and multi-period decision making problems.

Three-way decision theory provides a more general methods, which combine the universal decision-making thoughts and the intuitive information processing patterns together. Recently, Yao [46] further presented a useful sequential three-way decision theory for solving practical decision-making problems when information is unavailable and is acquired on demands with associated cost. Liang and Wang [47,49] introduced the three-way decisions into multi-attribute group decision making and designed an intelligent Dempster-Shafer theory based group sorting method.

(3) Multigranulation decision-theoretic rough set model

With the development of the economy and society, more and more complex uncertain decision-making problems come into people's daily life and social administration. Many existing single-granulation rough set models including decision-theoretic rough set model can be used to solve some of the uncertain decision problems. When facing high-dimensional, large-group and inconsistent multi-attribute decision-making problems, these single-granulation models have difficulties in approximating concepts and deriving rules. In [48], Qian et al. proposed a multigranulation decision-theoretical rough set model, which combined with the thoughts of multigranulation rough set [33] and decision-theoretic rough set [42], to solve the complex decision-making problems by fusing multiple relations induced from the decision systems.

Three typical multigranulation decision-theoretic rough set models including the first two extreme models and the last moderate model, are defined for obtaining better decision results.

– Optimistic multigranulation decision-theoretic rough set model (OMGDTRS)
  In decision analysis, an "optimistic" decision maker is used to express the idea that the lower approximation of a concept only needs at leat one granular structure to satisfy with the inclusion condition. Then, the *min* operator is used to fuse the conditional possibilities of different knowledge granularities.

- Pessimistic multigranulation decision-theoretic rough set model (PMGDTRS)
  In decision analysis, a "pessimistic" decision maker often have the decision strategy, seeking common ground while eliminating differences, which is a conservative decision strategy. Then, the *max* operator is used to fuse the conditional possibilities of different knowledge granularities.
- Mean multigranulation decision-theoretic rough set model (MMGDTRS)
  In decision analysis, most of decision makers are with the "moderate" attitude. Then, the arithmetic *mean* operator is used to fuse the conditional possibilities of different knowledge granularities for approximating the objective concept.

MGDTRS can derive many existing rough set models when the parameters satisfy special constraints. It provides a new perspective for rough decision analysis.

(4) Rough set based multi-attribute group decision-making model

Multi-attribute group decision making (MAGDM) is an important component of the modern decision science. The theories and methods of MAGDM have been widely applied in economic, management and many other fields. In many practical fields, the experts in group provide not only the decision matrixes, but also the primary sorting for alternatives by intuition. Many existing multi-attribute group decision-making methods focus on integrating the evaluations under multiple attributes and multiple experts, the intuition judgments of experts are, however, not considered. In Ref. [49], Liang and Wang proposed a rough set based multi-attribute group decision model (RS-MAGDM) to take a full consideration of the primary sorting.

A multi-attribute group decision system is denoted as $S^k = (U, C \cup \{d^k\}, f^k, V^k)$, where $U = \{x_1, x_2, ..., x_n\}$ is a set of alternatives, $C = \{c_1, c_2, ..., c_m\}$ is a set of attributes. A group of experts $E = \{e_1, e_2, ..., e_l\}$ evaluated the alternatives under the attribute set C and d. $f^k : U \times C \cup d^k \to V^k$ is an evaluation function.

The main processes of RS-MAGDM are shown as follows.

- Determination of weights of attributes
  The local weight of c_j^k is determined by the consistent degree that measures the dominance granulation structures between the attribute c_j^k and the primary decision attribute d^j, i.e., $w_j^k = sim(\succeq_{c_j^k}, \succeq_{d^k})$. The overall weight of c_j, w_j, is computed from the average of w_j^k.
- Determination of weights of experts
  The decision similarity degree $sim(e_k, e_{k'})$ between e_k and $e_{k'}$ is calculated by measuring the close degree between the structures of rough sets. The representative expert e^* is chosen for the most consistency with the other experts. Then, ρ_k, the weight of e_k, is determined by calculating the similarity degree between e_k and the representative expert e^*.
- Aggregation of decision information

The overall evaluation value of x is computed by integrating the values under multiple attributes and multiple experts. Then the corresponding decision tasks can be accomplished on the basis of overall values.

Moreover, based on dominance granular structures of DRS, Pang and Liang [50] designed six indices for evaluating the results of multi-attribute group decision making. In [50], three key evaluation indices C^k, T^k and U^k were proposed for measuring the decision consistency, closeness and uniformity of an expert in multi-attribute group decision making. Based on the three indices for individual experts, the other three indices C, T and K were designed for measuring the group consistency, group closeness, and group uniformity degrees.

RS-MAGDM introduces the intuitive judgements of experts into the process of the multi-attribute group decision making. The weights of attributes and experts are calculated from the dominance granular structures in decision matrix and the rough approximation structures of expert's primary judgements. The introduction of intuitive judgments and the natural computation of weights does brighten the model for solving the general multi-attribute group decision-making problems.

3 Concluding Remarks and Future Perspectives

The focal point of interest in the paper is to introduce four kinds of decision-closely-related rough set models. These models and methods have received much attention of the researchers from the fields of artificial intelligence, decision analysis, mathematics and others. They have also been successfully applied to solve many practical decision-making problems. Nevertheless, there still exist some challenging issues in the theoretical and practical researches of decision-oriented rough sets.

- Hybrid decision systems are common in practise. It is worth the effort to construct suitable decision-oriented rough set models for solving the hybrid decision-making problems.
- The fusion of rough set with other decision methods can open a door for the complex decision making.
- Many practical decision-making problems emerge with the characteristics of dynamic, large-sale of alternatives and high-dimensional attributes. It is another research hotspot to design effective and fast algorithms for this kind of problems.
- It is the objective of practitioners to apply diverse rough set models to real decision processes for obtaining reasonable and better decision results.

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