Variable Precision Multi-granulation Rough Set

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***Abstract*—Multi-granulation rough set model is an important generalization of Pawlak's rough set model. To enhance its capability of dealing with noisy data, in this paper, we proposed the variable precision multi-granulation rough set model and the variable precision weighted multi-granulation rough set model by loosing the requirement of accuracy on each granulation. Furthermore, we gave some important properties, and analyzed the relationships between existing multi-granulation rough set models and the proposed models.**

***Keywords*-Granular computing; Rough set; Multi-granulation; Variable precision**

I. Introduction

Rough set theory, originated by Pawlak [2], [4], [5], has become a popular mathematical framework for pattern recognition, image processing, feature selection, data mining and knowledge discovery process. Pawlak's rough set model is mainly concerned with the approximation of sets described by a single binary relation on the universe. Qian et. al. [6], [7] extends Pawlak's rough set model to a multi-granulation rough set model (MGRS), where the set approximations are defined by using multi equivalence relations. These equivalence relations can be chosen according to a user's requirements or targets of problem solving. Therefore, multi-granulation rough set model is an more effective approach for problem solving than Pawlak's rough set in the context of multi granulations. And it must be noted that fuzzy rough sets proposed by Dubois [1] is hierarchically multigranule because an alpha-cut of fuzzy rough set gives a partition.

Later, Xu et. al. [10] proposed a more generalized and logical one. The multi-granulation rough set model proposed in [6], [7], [8] are only special ones. Yang et. al. [11] generalized the multi-granulation rough set into fuzzy environment. A family of fuzzy T-similarity relations are used to define the optimistic and pessimistic fuzzy rough sets respectively. From a multi-granulation view, Liang et. al.[3] given an efficient rough feature selection algorithms. Moreover, researchers have proposed several generalized rough set models to process data with noise. Yao et al.[12], [13] gave the model of decision-theoretic rough sets. Ziarko [14] proposed variable precision rough set moldel.

To generalize multi-granulation rough sets model to process data with noise, in this paper, we proposed a new generalization of the variable precision multi-granulation rough set model. It allows for a controlled degree of misclassification in its formalism which, in turn, leads to more general notions of set approximations.

II. Preliminaries

In this section, we review some basic concepts on Pawlak's rough set model and multi-granulation rough set model.

An information system is a 4-tuple $S = \{U, A, V, f\}$ (for short $S = \{U, A\})$, where U is a non-empty and finite set of objects, called a universe, and A is a non-empty and finite set of attributes, V_a is the domain of the attribute a, $V = \bigcup_{a\in A} V_a$ and $f : U \times A \to V$ is a function $f(x, a) \in V_a$ for each $a \in A$.

An indiscernibility relation $R_B = \{(x, y) \in U \times U \mid a(x) = a(y), \forall a \in B\}$ was determined by a non-empty subset $B \subseteq A$. $U/R_B = \{[x]_B \mid x \in U\}$ (just as U/B) indicates the partitions of U resulted from R_B, where $[x]_B$ denotes the equivalence class determined by x with respect to B, i.e., $[x]_B = \{y \in U \mid (x, y) \in R_B\}$.

Furthermore, for any $Y \subseteq U$, $(\overline{B}(Y), \underline{B}(Y))$ is defined as the rough set of Y with respect to B, where the lower approximation $\underline{B}(Y)$ and the upper approximation $\overline{B}(Y)$ of Y are indicated by

$$\underline{B}(Y) = \{x|[x]_B \subseteq Y\},$$
$$\overline{B}(Y) = \{x|[x]_B \cap Y \neq \emptyset\}.$$

Multi-granulation rough set model is an expanding of Pawlak rough set model[7], [8], [10]. A target concept is approximated through several different equivalence relationship in multi-granulation rough set model. In the following, we review some common models.

The optimistic and pessimistic multi-granulation roughs set model are defined as following.

Definition 1: [8] Let $S = (U, A, V, f)$ be an information system, $X \subseteq U$ and $P = \{P_i \subseteq A|P_i \cap P_j = \emptyset (i \neq j), i, j \leq l\}$. The pessimistic lower and upper approximation sets of X with respect to P can be defined as follows:

$$\underline{OM}(X) = \{x \in U| \vee ([x]_{P_i} \subseteq X), i \leq l\},$$
$$\overline{OM}(X) = \{x \in U| \wedge ([x]_{P_i} \cap X \neq \emptyset), i \leq l\}.$$

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X is definable if and only if $\underline{OM}(X) = \overline{OM}(X)$, otherwise X is rough if and only if $\underline{OM}(X) \neq \overline{OM}(X)$. $\underline{OM}(X)$ and $\overline{OM}(X)$ are called,respectively, optimistic lower and upper approximation sets.

Definition 2: [9] Let $S = (U, A, V, f)$ be an information system, $X \subseteq U$ and $P = \{P_i \subseteq A | P_i \cap P_j = \emptyset (i \neq j), i, j \leq l\}$. The pessimistic lower and upper approximation sets of X with respect to P can be defined as follows:

$$\underline{PM}(X) = \{x \in U | \wedge ([x]_{P_i} \subseteq X), i \leq l\},$$
$$\overline{PM}(X) = \{x \in U | \vee ([x]_{P_i} \cap X \neq \emptyset), i \leq l\}.$$

X is definable if and only if $\underline{PM}(X) = \overline{PM}(X)$, otherwise X is rough if and only if $\underline{PM}(X) \neq \overline{PM}(X)$. $\underline{PM}(X)$ and $\overline{PM}(X)$ are called,respectively, pessimistic lower and upper approximation sets.

Definition 3: [10] Let $S = (U, A)$ be an information system, $X \subseteq U$ and $P = P_i \subseteq A | P_i \cap P_j = (i \neq j), i, j \leq l$. Characteristic function $S^X_{P_i}(x)$, describing the inclusion relation between the class $[x]_{P_i}$ and the concept X, is defined as follows:

$$S^X_{P_i}(x) = \begin{cases} 1, & [x]_{P_i} \subseteq X, \quad (1) \\ 0, & else. \quad (1') \end{cases}$$

Based on the definition, Xu et. al. proposed a new multi-granulation rough set model with a parameter $\beta \in (0.5, 1]$. In this model, the parameter determines how many granulations support a target concept included and the ones possibly describing the concept below the corresponding level are ignored. This model is presented in the definition below.

Definition 4: [10] Let $S = (U, A)$ be an information system, $X \subseteq U$ $P = \{P_i \in A | P_i \cap P_j = (i \neq j), i, j \leq l\}$. $S^X_{P_i}(x)$ is supporting characteristic function of x. For any $\beta \in (0.5, 1]$, the lower and upper approximation sets of X with respect to P are defined as follows.

$$\underline{P}(X)^{Xu}_\beta = \{x \in U | \frac{\sum_{i=1}^l S^X_{P_i}(x)}{l} \geq \beta\},$$
$$\overline{P}(X)^{Xu}_\beta = \{x \in U | \frac{\sum_{i=1}^l (1 - S^X_{P_i}(x))}{l} > 1 - \beta\}.$$

III. Variable precision multi-granulation rough set

In this section, the variable precision multi-granulation rough set model will be given by using the idea of partial including into rough approximation operators.

Definition 5: Let $S = (U, A)$ be information system, $X \subseteq U$ and $P = \{P_i \subseteq A | P_i \cap P_j = \emptyset (i \neq j), i, j \leq l\}$, then the lower and upper approximations of X with respect to P are defined as follows:

$$\underline{VP}(X)^\alpha_\beta = \{x \in U | \sum_{i=1}^l w^\alpha_i \times \mu^X_{P_i}(x) \geq \beta\},$$
$$\overline{VP}(X)^\alpha_\beta = \sim \underline{P}(\sim X)^\alpha_\beta,$$

where $\sim$ is means the complementary operation of a set,

$$\omega^\alpha_i = \begin{cases} \frac{1}{l}, & \alpha \leq \mu^X_{P_i}(x) \leq 1, \quad (2) \\ 0, & \mu^X_{P_i}(x) < \alpha. \quad (2') \end{cases}$$

In this definition, the parameter α determines the precision of every granulation which are used to approximate the target concept.

Proposition 1: Let $S = (U, A)$ be an information system, $X \subseteq U$ and $P = \{P_i \subseteq A | P_i \cap P_j = \emptyset (i \neq j), i, j \leq l\}$. If $\alpha = 1$, then

$$\underline{VP}(X)^1_\beta = \underline{P}(X)^{Xu}_\beta,$$
$$\overline{VP}(X)^1_\beta = \overline{P}(X)^{Xu}_\beta.$$

Proof: By the existing condition, we have that

$$\begin{aligned} \underline{VP}(X)^1_\beta &= \{x \in U | \sum_{i=1}^l w^1_i \times \mu^X_{P_i}(x) \geq \beta\} \\ &= \{x \in U | \sum_{i=1, \mu^X_{P_i}(x)=1}^l \tfrac{1}{l} \geq \beta\} \\ &= \underline{P}(X)^{Xu}_\beta, \end{aligned}$$

$$\begin{aligned} \overline{VP}(X)^1_\beta &= \sim \underline{P}(\sim X)^1_\beta \\ &= \sim \{x \in U | \sum_{i=1}^l w^1_i \times \mu^{\sim X}_{P_i}(x) \geq \beta\} \\ &= \sim \{x \in U | \sum_{i=1, \mu^{\sim X}_{P_i}(x)=1}^l \tfrac{1}{l} \geq \beta\} \\ &= \sim \{x \in U | \sum_{i=1, \mu^X_{P_i}(x)=0}^l \tfrac{1}{l} \geq \beta\} \\ &= \{x \in U | \sum_{i=1, \mu^X_{P_i}(x)\neq 0}^l \tfrac{1}{l} \geq \beta\} \\ &= \overline{P}(X)^{Xu}_\beta. \end{aligned}$$

■

Proposition 1 states that variable precision multi-granulation rough approximation is degraded as Xu's multi-granulation rough approximation, when the parameter $\alpha = 1$. That is to say, the proposed model is a more general multi-granulation rough set model.

Example 1: Let $S = (U, C)$, $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$, $X = \{x_1, x_2, x_6, x_8\}$, $U/P_1 = \{\{x_1, x_7\}, \{x_2, x_3, x_4, x_6, x_8\}, \{x_5\}\}$, $U/P_2 = \{\{x_1\}, \{x_2, x_3, x_6, x_8\}, \{x_4, x_5\}, \{x_7\}\}$.

By computing, we have that
$\mu^X_{P_1}(x_1) = 0.5$, $\mu^X_{P_2}(x_1) = 1$,
$\mu^X_{P_1}(x_2) = 0.6$, $\mu^X_{P_2}(x_2) = 0.75$,
$\mu^X_{P_1}(x_3) = 0.6$, $\mu^X_{P_2}(x_3) = 0$,
$\mu^X_{P_1}(x_4) = 0.6$, $\mu^X_{P_2}(x_4) = 0$,
$\mu^X_{P_1}(x_5) = 0$, $\mu^X_{P_2}(x_5) = 0$,
$\mu^X_{P_1}(x_6) = 0.6$, $\mu^X_{P_2}(x_6) = 0.75$,
$\mu^X_{P_1}(x_7) = 0.5$, $\mu^X_{P_2}(x_7) = 0$,
$\mu^X_{P_1}(x_8) = 0.6$, $\mu^X_{P_2}(x_8) = 0.75$.

Thus,
$\underline{VP}(X)^{0.3}_{0.3} = \{x_1, x_2, x_3, x_4, x_6, x_8\}$,
$\underline{VP}(X)^{0.7}_{0.3} = \{x_1, x_2, x_3, x_6, x_8\}$,
$\underline{VP}(X)^1_{0.3} = \{x_1\}$.

Furthermore, by computing, we have
$\underline{P}(X)^{Xu}_{0.3} = \{x_1\}$.
It is obvious that $\underline{VP}(X)^1_{0.3} = \underline{P}(X)^{Xu}_{0.3}$.
In the similar way, we easily obtain that
$\overline{VP}(X)^1_{0.3} = \overline{P}(X)^{Xu}_{0.3}$.

The following propositions will give some important properties of variable precision multi-granulation rough set model.

Proposition 2: For every $0.5 \leq \alpha \leq 1$, $0 < \beta \leq 1$, the following relationship are true:

$(1)\underline{VP}(X)^{\alpha}_{\beta} \supseteq \underline{VP}(X)^{1}_{\beta}$,
$(2)\overline{VP}(X)^{\alpha}_{\beta} \subseteq \overline{VP}(X)^{1}_{\beta}$,
$(3)BND_{VP}(X)^{\alpha}_{\beta} \subseteq BNR_{VP}(X)^{1}_{\beta}$,
$(4)NEG_{VP}(X)^{\alpha}_{\beta} \subseteq NEG_{VP}(X)^{1}_{\beta}$.

Proposition 3: For every $0.5 \leq \alpha \leq 1$, $0 < \beta \leq 1$, the following relationship are true:

$(1a)X \supseteq^{\alpha} \underline{VP}(X)^{\beta}_{\alpha}$;
$(1b)\overline{VP}(X)^{\alpha}_{\beta} \subseteq \underline{VP}(X)^{\alpha}_{\beta}$;
$(2a)\underline{VP}(\emptyset)^{\alpha}_{\beta} = \overline{VP}(\emptyset)^{\alpha}_{\beta} = \emptyset$;
$(2b)\underline{VP}(U)^{\alpha}_{\beta} = \overline{VP}(U)^{\alpha}_{\beta} = U$;
$(3a)X \subseteq Y \Leftrightarrow \underline{VP}(X)^{\alpha}_{\beta} \subseteq \underline{VP}(Y)^{\alpha}_{\beta}$;
$(3b)X \subseteq Y \Leftrightarrow \overline{VP}(X)^{\alpha}_{\beta} \subseteq \overline{VP}(Y)^{\alpha}_{\beta}$;
$(4a)\underline{VP}(X \cap Y)^{\alpha}_{\beta} \subseteq \underline{VP}(X)^{\alpha}_{\beta} \cap \underline{VP}(Y)^{\alpha}_{\beta}$;
$(4b)\overline{VP}(X \cup Y)^{\alpha}_{\beta} \subseteq \overline{VP}(X)^{\alpha}_{\beta} \cup \overline{VP}(Y)^{\alpha}_{\beta}$;
$(5a)\underline{VP}(X \cup Y)^{\alpha}_{\beta} \supseteq \underline{VP}(X)^{\alpha}_{\beta} \cup \underline{VP}(Y)^{\alpha}_{\beta}$;
$(5b)\overline{VP}(X \cap Y)^{\alpha}_{\beta} \supseteq \underline{VP}(X)^{\alpha}_{\beta} \cap \overline{VP}(Y)^{\alpha}_{\beta}$.

Proof: $(1a)$ To demonstrate that $X \supseteq^{\alpha} \underline{VP}^{\alpha}_{\beta}(X)$ it suffices to show that for any two elementary sets E_1, E_2 if $\mu^{E_1}_{P_i}(x) \geq \alpha$ and $\mu^{E_2}_{P_i}(x) \geq \alpha$ then $\mu^{E_1 \cup E_2}_{P_i}(x) \geq \alpha$.

We suppose that $\mu^{E_1}_{P_i}(x) = \frac{|[x]_{P_i} \cap E_1|}{|[x]_{P_i}|} \geq \alpha$ and $\mu^{E_2}_{P_i}(x) = \frac{|[x]_{P_i} \cap E_2|}{|[x]_{P_i}|} \geq \alpha$. Thus
$\mu^{E_1 \cup E_2}_{P_i}(x) = \frac{|[x]_{P_i} \cap (E_1 \cup E_2)|}{|[x]_{P_i}|} \geq \mu^{E_1}_{P_i}(x) \geq \alpha$
$(2a)$ From the definition of lower and upper approximations
$\underline{VP}(\emptyset)^{\alpha}_{\beta} = \{x \in U | \sum_{i=1}^{l} w^{\alpha}_i(\emptyset) \times \mu^{\emptyset}_{P_i}(x) = 0 \geq \beta\} = \emptyset$,
$\underline{VP}(U)^{\alpha}_{\beta} = \{x \in U | \sum_{i=1}^{l} w^{\alpha}_i(U) \times \mu^{U}_{P_i}(x) = 1 \geq \beta\} = U$.
In the similar way, we can prove $\overline{VP}(\emptyset)^{\alpha}_{\beta} = \emptyset$ and $\overline{VP}(U)^{\alpha}_{\beta} = U$.
(3a) For any $x \in \underline{VP}(X)^{\alpha}_{\beta}$, we have $\sum_{i=1}^{l} w_i(X)^{\alpha} \times \mu^{X}_{P_i}(x) \geq \beta$. Since $X \subseteq Y$, one can have $\mu^{X}_{P_i}(x) \leq \mu^{Y}_{P_i}(x)$. Then, $\sum_{i=1}^{l} w^{\alpha}_i(Y) \times \mu^{Y}_{P_i}(x) \geq \sum_{i=1}^{l} w^{\alpha}_i(X) \times \mu^{X}_{P_i}(x) \geq \beta$.
So, $x \in \underline{VP}(Y)^{\alpha}_{\beta}$. Therefore, this item is proved and item(3b) can be proved similarly.
$(4a)$ From the existing conditions, we have
$x \in P(X \cap Y)^{\alpha}_{\beta}$
$\Leftrightarrow \sum_{i=1}^{l} w^{\alpha}_i(X \cap Y) \times \mu^{X \cap Y}_{P_i}(x) = \sum_{i=1}^{l} w^{\alpha}_i(X \cap Y) \times \frac{|[x]_{p_i} \cap (X \cap Y)|}{|[x]_{p_i}|} \geq \beta$
$\Leftrightarrow \sum_{i=1}^{l} w^{\alpha}_i(X) \times \mu^{X}_{P_i}(x) \geq \beta$ and $\sum_{i=1}^{l} w^{\alpha}_i(X) \times \mu^{Y}_{P_i}(x) \geq \beta$
$\Leftrightarrow x \in \underline{VP}(X)$ and $x \in \underline{VP}(Y)$
$\Leftrightarrow x \in \underline{VP}(X) \cap \underline{VP}(Y)$
$(4b)$ From the duality property, the item can be proved by (4a).

Furthermore, by properties (3a) and (3b), (5a) and (5b) can be easily proved. ■

IV. Variable Precision Weighted Multi-Granulation Rough Set

In this section, a weighed multi-granulation rough approximations is first proposed, and its variable precision edition is introduced. Several important properties of the variable precision edition are given finally.

Definition 6: Let $S = (U, A)$ be an information system, $X \subseteq U$ and $P = \{P_i \subseteq A | P_i \cap P_j = \emptyset (i \neq j), i, j \leq l\}$, then variable precision weighted lower and upper approximations of X with respect to P are defined as follows:

$$\underline{WP}(X)_{\beta} = \{x \in U | \sum_{i=1}^{l} \eta_i \times \mu^{X}_{P_i}(x) \geq \beta\},$$
$$\overline{WP}(X)_{\beta} = \sim \underline{WP}(\sim X)_{\beta},$$

where, $\sim$ is means the complementary operation of a set,

$$\eta_i = \begin{cases} \dfrac{|[x]_{p_i}|}{\sum_{i=1}^{l} |[x]_{p_i}|}, & \mu^{X}_{P_i}(x) = 1, \quad (3a) \\ 0, & \mu^{X}_{P_i}(x) < 1. \quad (3b) \end{cases}$$

In this definition, the parameter α determines the accuracy of every granulation which are used to approximate the target concept.

Proposition 4: Let $S = (U, A)$ be an information system, $X \subseteq U$ and $P = \{P_i \subseteq A | P_i \cap P_j = \emptyset (i \neq j), i, j \leq l\}$. If $\alpha = 1$, $\beta = \xi$, then

$$\underline{WP}(X)_{\xi} = \underline{OM}(X),$$
$$\overline{WP}(X)_{\xi} = \sim \underline{OM}(\sim X),$$

where $\underline{WP}(X)_{\xi} = \{x \in U | \sum_{i=1}^{l} \eta_i \times \mu^{X}_{P_i}(x) \geq 0\} - \{x \in U | \sum_{i=1}^{l} \eta_i \times \mu^{X}_{P_i}(x) = 0\}$.

Proof: By the existing condition, we have that

$$\begin{aligned} \underline{WP}(X)_{\xi} &= \{x \in U | \sum_{i=1}^{l} \eta_i \times \mu^{X}_{P_i}(x) \geq \xi\} \\ &= \{x \in U | \sum_{i=1, \mu^{X}_{P_i}(x)=1}^{l} \frac{|[x]_{p_i}|}{\sum_{i=1}^{l} |[x]_{p_i}|} \geq \xi > 0\} \\ &= \{x \in U | \vee ([x]_{P_i} \subseteq X), i \leq l\} \\ &= \underline{OM}(X), \end{aligned}$$

$$\begin{aligned} \overline{WP}(X)_{\xi} &= \sim \underline{P}(\sim X)_{\beta} \\ &= \sim \{x \in U | \sum_{i=1}^{l} \eta_i \times \mu^{\sim X}_{P_i}(x) \geq \xi > 0\} \\ &= \sim \{x \in U | \sum_{i=1, \mu^{\sim X}_{P_i}(x)=1}^{l} \frac{|[x]_{p_i}|}{\sum_{i=1}^{l} |[x]_{p_i}|} > 0\} \end{aligned}$$

$=\sim \{x \in U | \sum_{i=1,\mu_{P_i}^X(x)=0}^{l} \frac{|[x]_{P_i}|}{\sum_{i=1}^{l}|[x]_{P_i}|} > 0\}$
$= \{x \in U | \sum_{i=1,\mu_{P_i}^X(x)=0}^{l} \frac{|[x]_{P_i}|}{\sum_{i=1}^{l}|[x]_{P_i}|} = 0\}$
$= \{x \in U | \wedge ([x]_{P_i} \cap X \neq \emptyset), i \leq l\}$
$= \overline{OM}(X).$ ■

Proposition 5: Let $S = (U, A)$ be an information system, $X \subseteq U$ and $P = \{P_i \subseteq A | P_i \cap P_j = \emptyset (i \neq j), i, j \leq l\}$. If $\alpha = 1$, $\beta = \xi$, then

$$\underline{WP}(X)_1 = \underline{PM}(X),$$
$$\overline{WP}(X)_1 =\sim \underline{PM}(\sim X).$$

Proof: By the existing condition, we have that

$$\begin{aligned}\underline{WP}(X)_1 &= \{x \in U | \textstyle\sum_{i=1}^{l} \eta_i \times \mu_{P_i}^X(x) \geq 1\} \\ &= \{x \in U | \textstyle\sum_{i=1,\mu_{P_i}^X(x)=1}^{l} \frac{|[x]_{P_i}|}{\sum_{i=1}^{l}|[x]_{P_i}|} \geq 1\} \\ &= \{x \in U | \wedge ([x]_{P_i} \subseteq X), i \leq l\} \\ &= \underline{PM}(X),\end{aligned}$$

$$\begin{aligned}\overline{WP}(X)_\beta &=\sim \underline{WP}(\sim X)_1 \\ &=\sim \{x \in U | \textstyle\sum_{i=1}^{l} \eta_i^1 \times \mu_{P_i}^{\sim X}(x) \geq 1\} \\ &=\sim \{x \in U | \textstyle\sum_{i=1,\mu_{P_i}^{\sim X}(x)=1}^{l} \frac{|[x]_{P_i}|}{\sum_{i=1}^{l}|[x]_{P_i}|} \geq 1\} \\ &=\sim \{x \in U | \textstyle\sum_{i=1,\mu_{P_i}^X(x)=0}^{l} \frac{|[x]_{P_i}|}{\sum_{i=1}^{l}|[x]_{P_i}|} \geq 1\} \\ &= \{x \in U | 0 < \textstyle\sum_{i=1,\mu_{P_i}^X(x)\neq 0}^{l} \frac{|[x]_{P_i}|}{\sum_{i=1}^{l}|[x]_{P_i}|} < 1\} \\ &= \{x \in U | \vee ([x]_{P_i} \cap X \neq \emptyset), i \leq l\} \\ &= \overline{PM}(X).\end{aligned}$$ ■

Propositions 4 and 5 state that optimistic and pessimistic multi-granulation rough set models are special case of weighted multi-granulation rough set model. In the following, we further introduce the variable precision edition of the weighted multi-granulation rough set.

Example 2: (Continued from Example 1) By means of Example 1, we have that

$$\underline{WP}(X)_{0.3}^{0.3} = \{x_1, x_2, x_3, x_4, x_6, x_7, x_8\},$$
$$\underline{WP}(X)_{0.3}^{0.7} = \{x_1, x_2, x_3, x_6, x_8\},$$
$$\underline{WP}(X)_{0.3}^{1} = \{x_1\}.$$

Furthermore, by computing, we have

$$\underline{WP}(X) = \{x_1\}.$$

It is obvious that $\underline{OM}(X) = \underline{WP}(X)_{0.3}^1$.
In a similar way, we have that $\overline{OM}(X) = \overline{WP}(X)_{0.3}^1$.

Definition 7: Let $S = (U, A)$ be an information system, $X \subseteq U$ and $P = \{P_i \subseteq A | P_i \cap P_j = \emptyset (i \neq j), i, j \leq l\}$, then variable precision weighted lower and upper approximations of X with respect to P are defined as follows:

$$\underline{VWP}(X)_\beta^\alpha = \{x \in U | \textstyle\sum_{i=1}^{l} \eta_i^\alpha \times \mu_{P_i}^X(x) \geq \beta\},$$
$$\overline{VWP}(X)_\beta^\alpha =\sim \underline{VWP}(\sim X)_\beta^\alpha,$$

where

$$\eta_i^\alpha = \begin{cases} \frac{|[x]_{P_i}|}{\sum_{i=1}^{l}|[x]_{P_i}|}, & \alpha \leq \mu_{P_i}^X(x) \leq 1, \quad (4) \\ 0, & \mu_{P_i}^X(x) < \alpha. \quad (4') \end{cases}$$

Proposition 6: Let $S = (U, A)$ be an information system, $X \subseteq U$ and $P = \{P_i \subseteq A | P_i \cap P_j = \emptyset (i \neq j), i, j \leq l\}$. If $\alpha = 1$, then

$$\underline{VWP}(X)_\beta^1 = \underline{WP}(X)_\beta,$$
$$\overline{VWP}(X)_\beta^1 = \overline{WP}(X)_\beta.$$

In the following, some properties of variable precision weighted multi-granulation rough set model will be introduced.

Proposition 7: For every $0.5 \leq \alpha \leq 1$, $0 < \beta \leq 1$, the following relationship are true:

$(1)\underline{VWP}(X)_\beta^\alpha \supseteq \underline{VWP}(X)_\beta^1,$
$(2)\overline{VWP}(X)_\beta^\alpha \subseteq \overline{VWP}(X)_\beta^1,$
$(3)BND_{VWP}(X)_\beta^\alpha \subseteq BNR_{VWP}(X)_\beta^1,$
$(4)NEG_{VWP}(X)_\beta^\alpha \subseteq NEG_{VWP}(X)_\beta^1.$

Proposition 8: For every $0.5 \leq \alpha \leq 1$, $0 < \beta \leq 1$, the following relationship are true:

$(1a)X \supseteq^\alpha \underline{VWP}(X)_\beta^\alpha;$
$(1b)\overline{VWP}(X)_\beta^\alpha \subseteq \underline{VWP}(X)_\beta^\alpha;$
$(2a)\underline{VWP}(\emptyset)_\beta^\alpha = \overline{VWP}(\emptyset)_\beta^\alpha = \emptyset;$
$(2b)\underline{VWP}(U)_\beta^\alpha = \overline{VWP}(U)_\beta^\alpha = U;$
$(3a)X \subseteq Y \Leftrightarrow \underline{VWP}(X)_\beta^\alpha \subseteq \underline{VWP}(Y)_\beta^\alpha;$
$(3b)X \subseteq Y \Leftrightarrow \overline{VWP}(X)_\beta^\alpha \subseteq \overline{VWP}(Y)_\beta^\alpha;$
$(4a)\underline{VWP}(X \cap Y)_\beta^\alpha \subseteq \underline{VWP}(X)_\beta^\alpha \cap \underline{VWP}(Y)_\beta^\alpha;$
$(4b)\overline{VWP}(X \cup Y)_\beta^\alpha \subseteq \overline{VWP}(X)_\beta^\alpha \cup \overline{VWP}(Y)_\beta^\alpha;$
$(5a)\underline{VWP}(X \cup Y)_\beta^\alpha \supseteq \underline{VWP}(X)_\beta^\alpha \cup \underline{VWP}(Y)_\beta^\alpha;$
$(5b)\overline{VWP}(X \cap Y)_\beta^\alpha \supseteq \underline{VWP}(X)_\beta^\alpha \cap \overline{VWP}(Y)_\beta^\alpha.$

In similar with Proposition 3, we easily prove this proposition.

V. CONCLUSION

In this paper, we proposed the variable precision multi-granulation rough set model based on the generalized multi-granulation rough set in [10]. Then we gave a weighted multi-granulation rough set model by weighted the accuracy of objects on each granulation, and introduced its variable precision edition through loosing the requirement of accuracy on each granulation. Finally, we analyzed some important properties, and studied the relationships between existing multi-granulation rough set models and the proposed models. These proposed models generalize the multi-granulation rough set approach, and are helpful to enhance its capability of dealing with noisy data.

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REFERENCES

[1] D. Dubois, H. Prade, Rough fuzzy sets and fuzzy rough sets, International Journal of General Systems 17 (1990) 191-208.

[2] J. Komorowski, Z. Pawlak, L. Polkowski, A. Skowron, Rough sets: a tutorial. In: S.K. Pal, A. Skowron (Eds.), Rough Fuzzy Hybridization: A New Trend in Decision-Making, Springer-Verlag, Singapore, 1999, pp. 3-98.

[3] J.Y. Liang, F. Wang, Y.H. Qian, An efficient rough feature selection algorithm with a multi-granulation view, International Journal of Approximate Reasoning (2012), http://dx.doi.org/10.1016/j.ijar.2012.02.004

[4] Z. Pawlak, Rough sets. International Journal of Computer and Information Sciences 11 (1982) 341-356.

[5] Z. Pawlak, Rough Sets: Theoretical Aspects of Reasoning about Data, Kluwer Academic Publishers, Boston, 1991.

[6] Y.H. Qian, J.Y. Liang, Rough Set Method Based on Multi-granulations: The 5th IEEE International Conference on Cognitive Informatics, Beijing, China, 2006, 297-304.

[7] Y.H. Qian, J.Y. Liang, C.Y. Dang, Incomplete Multigranulation Rough Set. IEEE Transactions on Systems, Man and Sybernetics-Part A: Systems and Human 40 (2) (2010) 420-431.

[8] Y.H. Qian, J.Y. Liang, Y.Y. Yao, C.Y. Dang, MGRS: A multi-granulation rough set. Information Sciences 180 (6) (2010) 949-970.

[9] Y.H. Qian, J.Y. Liang, W. Wei, Pessimistic rough decision. In: 2nd International Workshop on Rough Sets Theory, (2010) 19-21.

[10] W.H. Xu, X.T. Zhang, Q.R. Wang, A Generalized Multi-granulation Rough Set Approach. ICIC 2011, LNBI 6840, pp. 681-689, 2012.

[11] X.B. Yang, X.N. Song, H.L. Dou, J.Y. Yang, Multi-granulation rough set: from crisp to fuzzy case, Annals of Fuzzy Mathematics and Informatics 1 (1) (2011) 55-70.

[12] Y.Y. Yao, S.K.M. Wong, P. Lingras. A decision-theoretic rough set model, in: Z.W. Ras, M. Zemankova and M.L. Emrich (Eds.), Methodologies for Intelligent Systems, 5, North-Holland, New York, (1990) 17-24.

[13] Y.Y. Yao, Y. Zhao, Attribute reduction in decision-theoretic rough set models, Information Sciences 178 (17) (2008) 3356-3373.

[14] W. Ziarko, Variable precision rough set model, Journal of Computer and System Sciences 46 (1993) 39-59.