Evaluation Method for Decision Rule Sets

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Abstract. In this paper, a decision table in rough set theory is classified into three types according to its consistency. Three parameters α (whole certainty measure), β (whole consistency measure) and γ (whole support measure) are introduced to evaluate the performance of a decision rule set induced from a decision table. For three types of decision tables, the dependency of the parameters upon condition/decision granulation is analyzed. The parameters can be used to construct an evaluation function in favor of selecting a better one from some different rule acquiring methods for real decision problems.

Keywords: Rough set theory, decision table, decision rule, knowledge granulation, decision evaluation.

1 Introduction

Recently, rough set theory proposed by Pawlak in [1] has become a popular mathematical framework for pattern recognition, image processing, feature selection, neuro computing, conflict analysis, decision support, data mining and knowledge discovery process from large data sets [2-7]. For decision problems, by various kinds of reduct techniques, a set of decision rules can be generated from a decision table for classification or prediction [8-10].

In recent years, how to evaluate the performance of a decision rule has been becoming a very important issue in rough set theory[11-16]. In fact, a set of decision rules can be generated from a decision table by adopting any kind of reduction methods. In [11], Yao proposed several evaluation criterions for decision rules such as the generality, the absolute support, the change of support and the change of support, and so on. In [13], based on information entropy, Düntsch suggested some uncertainty measures of a decision rule, and proposed three criterions for model selection as well. In additional, several other measures such as certainty measure and support measure are often used to evaluate a decision rule [3, 7, 15]. However, because all of these measures are defined only for a single decision rule, they are unsuitable for measuring the whole performance of a rule set. Another two kinds of measures, the approximation accuracy for decision classes and the consistency degree for a decision table [1, 16], in some

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sense, could be regarded as measures for whole performance of all decision rules generated from a decision table. Nevertheless, the approximation accuracy and consistency degree have some limitations. For instance, the certainty and consistency of a rule set could not be well depicted by the approximation accuracy and consistency degree when their values achieve 0. As we know, the fact that approximation accuracy/consistency degree is equal to 0 only implies that there is no decision rule with the certainty 1 in the decision table. So the approximation accuracy and consistency degree of a decision table cannot give elaborate depictions of the certainty and consistency to a rule set.

This paper aims to find some criterions for evaluating the whole performance of a set of decision rules. In Section 2, some preliminary concepts such as indiscernibility relation, partition, partial relation of knowledge and decision table are briefly recalled. In Section 3, three parameters α, β and γ for evaluating a set of rules are introduced. The dependency of the parameters upon condition/decision granulation is analyzed. Section 4 concludes the paper.

2 Some Basic Concepts

An information system S is a pair (U, A), where U is a non-empty, finite set of objects called the universe and A is a non-empty, finite set of attributes, such that $a : U \rightarrow V_a$ for any $a \in A$, where V_a is called the domain of a.

Each non-empty subset $B \subseteq A$ determines an indiscernibility relation $R_B = \{(x, y) \in U \times U \mid a(x) = a(y), \forall a \in B\}$. The relation R_B partitions U into some equivalence classes $U/R_B = \{[x]_B \mid x \in U\}$, where $[x]_B = \{y \in U \mid (x, y) \in R_B\}$.

We define a partial relation $\preceq$ on the family $\{U/B \mid B \subseteq A\}$ as follows[17]: $U/P \preceq U/Q$ (or $U/Q \succeq U/P$), if and only if, for every $P_i \in U/P$, there exists $Q_j \in U/Q$ such that $P_i \subseteq Q_j$, where $U/P = \{P_1, P_2, \cdots, P_m\}$ and $U/Q = \{Q_1, Q_2, \cdots, Q_n\}$ are partitions induced by $P, Q \subseteq A$, respectively. In this case, we say that Q is coarser than P, or P is finer than Q. If $U/P \preceq U/Q$ and $U/P \neq U/Q$, we say Q is strictly coarser than P (or P is strictly finer than Q), denoted by $U/P \prec U/Q$ (or $U/Q \succ U/P$). It is clear that $U/P \prec U/Q$, if and only if, for every $X \in U/P$, there exists $Y \in U/Q$ such that $X \subseteq Y$, and there exist $X_0 \in U/P$, $Y_0 \in U/Q$ such that $X_0 \subset Y_0$.

A decision table is an information system $S = (U, C \cup D)$ with $C \cap D = \emptyset$, where C is called condition attribute set, and D is called decision attribute set. If $U/C \preceq U/D$, then $S = (U, C \cup D)$ is said to be consistent, otherwise it is inconsistent.

Definition 1. [1,16] *Let $S = (U, C \cup D)$ be a decision table, $X_i \in U/C$, $Y_j \in U/D$ and $X_i \cap Y_j \neq \emptyset$. By $des(X_i)$ and $des(Y_j)$, we denote the descriptions of the equivalence classes X_i and Y_j in the decision table S. A decision rule is formally defined as $Z_{ij} : des(X_i) \rightarrow des(Y_j)$.*

The certainty measure and support measure of a decision rule Z_{ij} are defined as $\mu(Z_{ij}) = |X_i \cap Y_j|/|X_i|$, $s(Z_{ij}) = |X_i \cap Y_j|/|U|$, where, by $|\cdot|$, we denote the cardinality of a set. It is clear that the values of $\mu(Z_{ij})$ and $s(Z_{ij})$ of a decision rule Z_{ij} fall into the interval $[\frac{1}{|U|}, 1]$.

By $|Z_{ij}|$, we denote the cardinality of the set $X_i \cap Y_j$, which is called the support number of the rule Z_{ij}. For convenience, by $a(x)$ $(a \in C)$ and $d(x)$ $(d \in D)$, we denote the values of the object x under the condition attribute a and the decision attribute d, respectively.

Definition 2. *Let $S = (U, C \cup D)$ be a decision table, $U/C = \{X_1, X_2, \cdots, X_m\}$, $U/D = \{Y_1, Y_2, \cdots, Y_n\}$. A condition class $X_i \in U/C$ is said to be consistent if $d(x) = d(y)$ for $\forall x, y \in X_i$ and $\forall d \in D$; a decision class $Y_j \in U/D$ is said to be converse consistent if $a(x) = a(y)$ for $\forall x, y \in Y_j$ and $\forall a \in C$.*

It is easy to see that a decision table $S = (U, C \cup D)$ is consistent if every condition class $X_i \in U/C$ is consistent.

Definition 3. *Let $S = (U, C \cup D)$ be a decision table, $U/C = \{X_1, X_2, \cdots, X_m\}$, $U/D = \{Y_1, Y_2, \cdots, Y_n\}$. S is said to be converse consistent, if every decision class $Y_j \in U/D$ is converse consistent, i.e., $U/D \preceq U/C$.*

A decision table is called a mixed decision table if it is neither consistent nor converse consistent.

$S = (U, C \cup D)$ is called to be restrict consistent (restrict converse consistent) if $U/C \prec U/D$ $(U/D \prec U/C)$.

Definition 4. [15,18] *Let $S = (U, A)$ be an information system, $U/A = \{R_1, R_2, \cdots, R_m\}$. The knowledge granulation of A is defined as*

$$G(A) = \frac{1}{|U|^2} \sum_{i=1}^{m} |R_i|^2. \tag{1}$$

Consequently, $G(C)$, $G(D)$ and $G(C \cup D)$ are called as the condition granulation, decision granulation and granulation of S, respectively.

3 Whole Performance Evaluation for a Rule Set

In rough set theory, several measures for a decision rule $Z_{ij} : des(X_i) \rightarrow des(Y_j)$ have been introduced in [1], such as certainty measure $\mu(X_i, Y_j) = |X_i \cap Y_j|/|X_i|$, support measure $s(X_i, Y_j) = |X_i \cap Y_j|/|U|$. However, because $\mu(X_i, Y_j)$ and $s(X_i, Y_j)$ are defined only for a single decision rule, they are unsuitable for measuring the whole performance of a rule set.

In [1], the approximation accuracy of a classification is introduced by Pawlak. Let $F = \{Y_1, Y_2, \cdots, Y_n\}$ be a classification of the universe U, and C a condition attribute set. $\underline{C}F = \{\underline{C}Y_1, \underline{C}Y_2, \cdots, \underline{C}Y_n\}$ and $\overline{C}F = \{\overline{C}Y_1, \overline{C}Y_2, \cdots, \overline{C}Y_n\}$ are called C-lower and C-upper approximations of F, where $\underline{C}Y_i = \bigcup\{x \in U \mid [x]_C \subseteq Y_i \in F\} (1 \leq i \leq n)$, $\overline{C}Y_i = \bigcup\{x \in U \mid [x]_C \cap Y_i \neq \emptyset, Y_i \in F\} (1 \leq i \leq n)$. The approximation accuracy of F by C is defined as $a_C(F) = \frac{\sum_{Y_i \in U/D} |\underline{C}Y_i|}{\sum_{Y_i \in U/D} |\overline{C}Y_i|}$. The approximation accuracy expresses the percentage of possible correct decisions when classifying objects employing the attribute set C. In a sense, $a_C(F)$ can be used to measure certainty of a decision table. The consistency degree of a decision table $S = (U, C \cup D)$, another measure in rough set theory, is defined as $c_C(D) = \frac{1}{|U|} \sum_{i=1}^{n} |\underline{C}Y_i|$. The consistency degree expresses the percentage of objects which can be correctly classified to decision classes of U/D by condition attribute set C. In a sense, $c_C(D)$ can be used to measure the consistency of a decision table.

Nevertheless, the certainty and consistency of a rule set could not be well depicted by approximation accuracy and consistency degree when their values achieve 0. Here, three new evaluation parameters α, β and γ are introduced to solve the problem.

Definition 5. *Let $S = (U, C \cup D)$ be a decision table, $RULE = \{Z_{ij} | Z_{ij} : des(X_i) \rightarrow des(Y_j), X_i \in U/C, Y_j \in U/D\}$. The certainty measure α of S is defined as*

$$\alpha(S) = \sum_{i=1}^{m} \sum_{j=1}^{n} s(Z_{ij})\mu(Z_{ij}) = \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{|X_i \cap Y_j|^2}{|U||X_i|}, \tag{2}$$

where $s(Z_{ij})$ and $\mu(Z_{ij})$ are the certainty measure and support measure of the rule Z_{ij}, respectively.

Although the parameter α is defined in the context of all decision rules from a decision table, it is also suitable to an arbitrary decision rule set as well.

Theorem 1 (Extremum). *Let $S = (U, C \cup D)$ be a decision table, $RULE = \{Z_{ij} | Z_{ij} : des(X_i) \rightarrow des(Y_j), X_i \in U/C, Y_j \in U/D\}$.*

(1) For every $Z_{ij} \in RULE$, if $\mu(Z_{ij}) = 1$, then the parameter α achieves its maximum value 1;

(2) If $m = 1$ and $n = |U|$, then parameter α achieves its minimum value $\frac{1}{|U|}$.

Remark. In fact, a decision table $S = (U, C \cup D)$ is consistent if and only if every decision rule from S is certain, i.e., its certainty measure is equal to 1. So, (1) of Theorem 1 shows that the parameter α achieves its maximum value 1 when S is consistent. (2) of Theorem 1 shows that α achieves its minimum value $\frac{1}{|U|}$ when we want to distinguish any two objects of U without any condition information.

Theorem 2. *Let $S_1 = (U, C_1 \cup D_1)$ and $S_2 = (U, C_2 \cup D_2)$ be two converse consistent decision tables. If $U/C_1 = U/C_2$, $U/D_2 \prec U/D_1$, then $\alpha(S_1) > \alpha(S_2)$.*

Proof. From $U/C_1 = U/C_2$ and the converse consistency of S_1 and S_2, it follows that there exist $X_p \in U/C_1$ and $Y_q \in U/D_1$ such that $Y_q \subseteq X_p$. By $U/D_2 \prec U/D_1$, there exist $Y_q^1, Y_q^2, \cdots, Y_q^s \in U/D_2$ $(s > 1)$ such that $Y_q = \bigcup_{k=1}^{s} Y_q^k$. In other words, the rule Z_{pq} in S_1 can be decomposed into a family of rules $Z_{pq}^1, Z_{pq}^2, \cdots, Z_{pq}^s$ in S_2. It is clear that $|Z_{pq}| = \sum_{k=1}^{s} |Z_{pq}^k|$. Therefore, $|Z_{pq}|^2 > \sum_{k=1}^{s} |Z_{pq}^k|^2$. Hence, by the definition of $\alpha(S)$, $\alpha(S_1) > \alpha(S_2)$.

Theorem 2 states that the certainty measure α of a converse consistent decision table decreases with its decision classes becoming finer.

Theorem 3. *Let $S_1 = (U, C_1 \cup D_1)$ and $S_2 = (U, C_2 \cup D_2)$ be two converse consistent decision tables. If $U/D_1 = U/D_2$, $U/C_2 \prec U/C_1$, then $\alpha(S_1) < \alpha(S_2)$.*

Proof. From $U/C_2 \prec U/C_1$, there exists $X_l \in U/C_1$ and an integer $s > 1$ such that $X_l = \bigcup_{k=1}^{s} X_l^k$, where $X_l^k \in U/C_2$. It is clear that $|X_l| = \sum_{k=1}^{s} |X_l^k|$, and therefore, $\frac{1}{|X_l|} < \frac{1}{|X_l^1|} + \frac{1}{|X_l^2|} + \cdots + \frac{1}{|X_l^s|}$.

Noticing that both S_1 and S_2 are converse consistent, we have $|Z_{lq}| = |Z_{lq}^k|$ $(k = 1, 2, \cdots, s)$. Hence, we have that

$$\begin{aligned}\alpha(S_1) &= \sum_{i=1}^{m}\sum_{j=1}^{n} s(Z_{ij})\mu(Z_{ij}) \\ &= \frac{1}{|U|}\sum_{i=1}^{l-1}\sum_{j=1}^{n}\frac{|Z_{ij}|^2}{|X_i|} + \frac{1}{|U|}\sum_{j=1}^{n}\frac{|Z_{lj}|^2}{|X_l|} + \frac{1}{|U|}\sum_{i=l+1}^{m}\sum_{j=1}^{n}\frac{|Z_{ij}|^2}{|X_i|} \\ &< \frac{1}{|U|}\sum_{i=1}^{l-1}\sum_{j=1}^{n}\frac{|Z_{ij}|^2}{|X_i|} + \frac{1}{|U|}\sum_{k=1}^{s}\sum_{j=1}^{n}\frac{|Z_{lj}|^2}{|X_l^k|} + \frac{1}{|U|}\sum_{i=l+1}^{m}\sum_{j=1}^{n}\frac{|Z_{ij}|^2}{|X_i|} \\ &= \alpha(S_2).\end{aligned}$$

Theorem 3 states that the certainty measure α of a converse consistent decision table increases with its condition classes becoming finer.

Definition 6. *Let $S = (U, C \cup D)$ be a decision table, $RULE = \{Z_{ij} | Z_{ij} : des(X_i) \rightarrow des(Y_j), X_i \in U/C, Y_j \in U/D\}$. The consistency measure β of S is defined as*

$$\beta(S) = \sum_{i=1}^{m} \frac{|X_i|}{|U|}[1 - \sum_{j=1}^{N_i} \mu(Z_{ij})(1 - \mu(Z_{ij}))], \tag{3}$$

where N_i is the number of decision rules determined by the condition class X_i, $\mu(Z_{ij})$ is the certainty measure of the rule Z_{ij}.

Although the parameter β is defined in the context of all decision rules from a decision table, it is also suitable to an arbitrary decision rule set as well.

Theorem 4 (Extremum). *Let $S = (U, C \cup D)$ be a decision table, $RULE = \{Z_{ij} | Z_{ij} : des(X_i) \rightarrow des(Y_j), X_i \in U/C, Y_j \in U/D\}$.*

(1) For every $Z_{ij} \in RULE$, if $\mu(Z_{ij}) = 1$, then the parameter β achieves its maximum value 1;

(2) For every $Z_{ij} \in RULE$, if $\mu(Z_{ij}) = \frac{1}{|U|}$, then the parameter β achieves its minimum value $\frac{1}{|U|}$.

It should be noted that the parameter β achieves its maximum 1 when $S = (U, C \cup D)$ be a consistent decision table.

Theorem 5. *Let $S_1 = (U, C_1 \cup D_1)$ and $S_2 = (U, C_2 \cup D_2)$ be two converse consistent decision tables or mixed decision tables. If $U/C_1 = U/C_2$, $U/D_2 \prec U/D_1$, then $\beta(S_1) > \beta(S_2)$.*

Proof. A mixed decision table S can be transformed into a converse consistent decision table S' via deleting all certainty decision rules. And it is clear that

$\beta(S) = \beta(S^{'})$. So, we only need to prove this theorem for converse consistent tables.

Since $U/C_1 = U/C_2$ and the converse consistency of S_1 and S_2, there exist $X_p \in U/C_1$ and $Y_q \in U/D_1$ such that $Y_q \subseteq X_p$. By $U/D_2 \prec U/D_1$, there exist $Y_q^1, Y_q^2, \cdots, Y_q^s \in U/D_2$ $(s > 1)$ such that $Y_q = \bigcup_{k=1}^{s} Y_q^k$. In other words, the rule Z_{pq} in S_1 can be decomposed into a family of rules $Z_{pq}^1, Z_{pq}^2, \cdots, Z_{pq}^s$ in S_2. It is clear that $|Z_{pq}| = \sum_{k=1}^{s} |Z_{pq}^k|$. Hence, we have that

$$\begin{aligned}\mu(Z_{pq})(1-\mu(Z_{pq})) &= \frac{|Z_{pq}||X_p|-|Z_{pq}|^2}{|X_p|^2} \\ &= \frac{|Z_{pq}^1+Z_{pq}^2+\cdots+Z_{pq}^s||X_p|-|Z_{pq}^1+Z_{pq}^2+\cdots+Z_{pq}^s|^2}{|X_p|^2} \\ &< \frac{|Z_{pq}^1+Z_{pq}^2+\cdots+Z_{pq}^s||X_p|-(|Z_{pq}^1|^2+|Z_{pq}^2|^2+\cdots+|Z_{pq}^s|^2)}{|X_p|^2} \\ &= \frac{|Z_{pq}^1||X_p|-|Z_{pq}^1|^2}{|X_p|^2} + \frac{|Z_{pq}^2||X_p|-|Z_{pq}^2|^2}{|X_p|^2} + \cdots + \frac{|Z_{pq}^s||X_p|-|Z_{pq}^s|^2}{|X_p|^2} \\ &= \sum_{k=1}^{s} \mu(Z_{pq}^k)(1-\mu(Z_{pq}^k)).\end{aligned}$$

Then, we can obtain that

$$\begin{aligned}\beta(S_1) &= \sum_{i=1}^{m} \frac{|X_i|}{|U|}[1-\sum_{j=1}^{N_i} \mu(Z_{ij})(1-\mu(Z_{ij}))] \\ &= \sum_{i=1}^{p-1} \frac{|X_i|}{|U|}[1-\sum_{j=1}^{N_i} \mu(Z_{ij})(1-\mu(Z_{ij}))] + \frac{|X_p|}{|U|}[1-\sum_{j=1}^{N_p} \mu(Z_{pj})(1- \\ &\quad \mu(Z_{pj}))] + \sum_{i=p+1}^{m} \frac{|X_i|}{|U|}[1-\sum_{j=1}^{N_i} \mu(Z_{ij})(1-\mu(Z_{ij}))] \\ &> \sum_{i=1}^{p-1} \frac{|X_i|}{|U|}[1-\sum_{j=1}^{N_i} \mu(Z_{ij})(1-\mu(Z_{ij}))] + \sum_{i=p+1}^{m} \frac{|X_i|}{|U|}[1-\sum_{j=1}^{N_i} \mu(Z_{ij})(1- \\ &\quad \mu(Z_{ij}))] + \frac{|X_p|}{|U|}[1-\sum_{k=1}^{s} \mu(Z_{pq}^k)(1-\mu(Z_{pq}^k)) - \sum_{j=1,j\neq q}^{N_i} \mu(Z_{pj})(1-\mu(Z_{pj}))] \\ &= \beta(S_2).\end{aligned}$$

Theorem 5 states that the consistency measure β of a mixed (or converse consistent) decision table decreases with its decision classes becoming finer.

Theorem 6. *Let $S_1 = (U, C_1 \cup D_1)$ and $S_2 = (U, C_2 \cup D_2)$ be two converse consistent decision tables or mixed decision tables. If $U/D_1 = U/D_2$, $U/C_2 \prec U/C_1$, then $\beta(S_1) < \beta(S_2)$.*

Proof. Similar to the proof of Theorem 5, it can be proved.

Theorem 6 states that the consistency measure β of a mixed (or converse consistent) decision table increases with its condition classes becoming finer.

Definition 7. *Let $S = (U, C \cup D)$ be a decision table, $RULE = \{Z_{ij}|Z_{ij} : des(X_i) \rightarrow des(Y_j), X_i \in U/C, Y_j \in U/D\}$. The support measure γ of S is defined as*

$$\gamma(S) = \sum_{i=1}^{m}\sum_{j=1}^{n} s^2(Z_{ij}) = \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{|X_i \cap Y_j|^2}{|U|^2}, \quad (4)$$

where $s(Z_{ij})$ is the support measure of the rule Z_{ij}.

Although the parameter γ is defined in the context of all decision rules from a decision table, it is suitable to an arbitrary decision rule set as well.

Theorem 7 (Extremum). *Let $S = (U, C \cup D)$ be a decision table, $RULE = \{Z_{ij} | Z_{ij} : des(X_i) \rightarrow des(Y_j), X_i \in U/C, Y_j \in U/D\}$.*

(1) If $m = n = 1$, then the parameter γ achieves its maximum value 1;

(2) If $m = |U|$ or $n = |U|$, then the parameter γ achieves its minimum value $\frac{1}{|U|}$.

Theorem 8. *Let $S_1 = (U, C_1 \cup D_1)$ and $S_2 = (U, C_2 \cup D_2)$ be two decision tables, then $\gamma(S_1) < \gamma(S_2)$, if and only if, $G(C_1 \cup D_1) < G(C_2 \cup D_2)$.*

Proof. Suppose $U/(C \cup D) = \{X_i \cap Y_j \mid X_i \cap Y_j \neq \emptyset, X_i \in U/C_1, Y_j \in U/D\}$, $RULE = \{Z_{ij} | Z_{ij} : X_i \rightarrow Y_j, X_i \in U/C, Y_j \in U/D\}$. From Definition 4 and $s(Z_{ij}) = \frac{|X_i \cap Y_j|}{|U|}$, it follows that

$$\begin{aligned} G(C \cup D) &= \frac{1}{|U|^2} \sum_{i=1}^{m} \sum_{j=1}^{n} |X_i \cap Y_j|^2 \\ &= \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\frac{|X_i \cap Y_j|}{|U|}\right)^2 = \sum_{i=1}^{m} \sum_{j=1}^{n} s^2(Z_{ij}) \\ &= \gamma(S). \end{aligned}$$

Therefore, $\gamma(S_1) < \gamma(S_2)$ if and only if $G(C_1 \cup D_1) < G(C_2 \cup D_2)$.

Theorem 8 states that the support measure γ of a decision table increases with the granulation of the decision table becoming bigger.

Theorem 9. *Let $S_1 = (U, C_1 \cup D_1)$ and $S_2 = (U, C_2 \cup D_2)$ be two converse consistent decision tables. If $U/C_1 = U/C_2$, $U/D_1 \prec U/D_2$, then $\gamma(S_1) < \gamma(S_2)$.*

Proof. Similar to Theorem 5, it can be proved.

Theorem 9 states that the support measure γ of a decision table decreases with its decision classes becoming finer.

4 Conclusions

In this paper, the limitations of the traditional measures are exemplified. Three parameters α, β and γ are introduced to measure the certainty, consistency and support of a rule set obtained from a decision table, respectively. For three types of decision tables (consistent, converse consistent and mixed), the dependency of parameters α, β and γ upon condition/decision granulation is analyzed.

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