Closed-Label Concept Lattice Based Rule Extraction Approach

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Abstract. Concept lattice is an effective tool for data analysis and extracting classification rules. However, the classical concept lattice often produce a lot of redundant rules. Closed-label concept lattice realizes reduction of concept intention, which can be used to extract fewer rules than the classical concept lattice. This paper presents a method for classification rules extraction based on the closed-label concept lattice. Examples show that the proposed method is effective for extracting more concise classification rules.

Keywords: Concept lattice, Closed-label concept lattice, Classification rules, ID3.

1 Introduction

Concept lattice theory was proposed by Wille[1] in 1982. In the concept lattice theory, the data for analysis are described by a formal context (U, A, I), which consists of universe U, attributes set A, and relation $I \in U \times A$. Based on the formal context, we can construct some formal concepts and the set of all the above formal concepts forms a concept lattice. The concepts are constituted by two parts: intension, which comprises all attributes shared by the objects, and extension, which consists of all objects belonging to the concept. The concept lattice reflects the relationship of generalization and specialization among concepts. It is an effective and intuitive way to represent, design and discover knowledge structures.

Concept lattice theory is a kind of important mathematical tool for conceptual knowledge processing and data analysis. It provides a theoretical framework for the discovery and design of concept hierarchies from relational information systems. Most of the researches on concept lattice focus on such topics as: construction of concept lattice[2,3], extended model of concept lattice[6], acquisition of rules[4,5], relationship with rough set[7-10], and attribute reduction[11-14]. To date, concept lattice has been applied to digital library, information retrieval, software engineering and other aspects[15-17].

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Classification rules mining is an important data mining task. The typical classification rule mining methods include decision tree, neural networks, rough sets and so on. Since concept lattice reflects the relationship between the formal concepts, it can be seen as a natural platform for rule extraction. Therefore the framework of the concept lattice is meaningful to discuss the issue of classification rules mining[4,5]. However, the classical concept lattice often produce a lot of redundant rules. In [6], we introduced a new lattice structure called closed-label concept lattice. In the new structure, all concepts are depicted by its intension reduction. Closed-label concept lattice realizes reduction of concept intention, which can be used to extract fewer rules than the classical concept lattice. The paper aims to present a method for extracting classification rules based on the closed-label concept lattice.

This paper is organized as follows. Basic definitions of concept lattice and closed label lattice are recalled in Section 2 and 3. In Section 4, closed-label concept lattice based rule extraction method is introduced. In Section 5, the proposed method compared with ID3 algorithm is given. Finally, some conclusions are given in Section 6.

2 Preliminaries

In this section, we review some basic concepts of concept lattice [1,18].

Definition 1. A formal context is a triplet (U, A, I), where $U = \{x_1, x_2, \ldots, x_n\}$ is a non-empty, finite set of objects called the universe of discourse,

$A = \{a_1, a_2, \ldots, a_m\}$ is a non-empty, finite set of attributes, and $I \subseteq U \times A$ is a binary relation between U and A, where $(x, a) \in I$ means that object x has attribute a.

In this paper, we assume that the binary relation I is regular, that is, it satisfies the following conditions: for any $(x, a) \in U \times A$,

(1) there exist $a_1, a_2 \in A$ such that $(x, a_1) \in I$ and $(x, a_2) \notin I$,

(2) there exist $x_1, x_2 \in U$ such that $(x_1, a) \in I$ and $(x_2, a) \notin I$.

For $X \subseteq U$ and $B \subseteq A$, we define

$$\begin{aligned} X^* &= \{a \in A : \forall x \in X, (x, a) \in I\}, \\ B' &= \{x \in U : \forall a \in B, (x, a) \in I\}. \end{aligned} \tag{1}$$

X^* is the maximal set of attributes shared by all objects in X. Similarly, B' is the maximal set of objects that have all attributes in B. For $x \in U$ and $a \in A$, we denote $x^* = \{x\}^*$ and $a' = \{a\}'$. Thus x^* is the set of attributes possessed by x, and a' is the set of objects having attribute a.

Definition 2. Let (U, A, I) be a formal context. A pair (X, B), with $X \subseteq U$ and $B \subseteq A$, is called a formal concept of the context (U, A, I) if $X^* = B$ and $B' = X$. The set of objects X and the set of attributes B are respectively called the extension and the intension of the formal concept (X, B).

In the paper, for a formal concept C, the extension noted as extension(C) and the intension noted as intension(C).

Table 1. A formal context

U	a	b	c	d	e
1	0	1	0	1	0
2	1	0	1	0	1
3	1	1	0	0	1
4	0	1	1	1	0
5	1	0	0	0	1

The set of all formal concepts forms a complete lattice called a concept lattice and is denoted by $L(U, A, I)$. The meet and join of the lattice are given by:

$$\begin{aligned}(X_1, B_1) \wedge (X_2, B_2) &= (X_1 \cap X_2, (B_1 \cup B_2)'^*),\\ (X_1, B_1) \vee (X_2, B_2) &= ((X_1 \cup X_2)^{*\prime}, B_1 \cap B_2).\end{aligned} \tag{2}$$

The corresponding partial order relation $\leq$ in the concept lattice $L(U, A, I)$ is given as follows: for $(X_1, B_1), (X_2, B_2) \in L(U, A, I)$,

$$(X_1, B_1) \leq (X_2, B_2) \Longleftrightarrow X_1 \subseteq X_2 \Longleftrightarrow B_1 \supseteq B_2. \tag{3}$$

A formal context can be represented by a table the rows of which are headed by the object names and the columns headed by the attribute names. A value 1 in row x and column a means that the object x has the attribute a.

In this paper,we use the table in [12] for example.

Example 1. Table 1 depicts an example of formal context $F = (U, A, I)$, where $U = \{1, 2, 3, 4, 5\}$, $A = \{a, b, c, d, e\}$, and for each $(x_i, a_j) \in U \times A$, $(x_i, a_j) \in I$ iff object x_i has value 1 in attribute a_j, i.e., $a_j(x_i) = 1$. Fig. 1 is the Hasse diagram of the concept lattice derived from Table 1.

3 Closed-Label Concept Lattice

In [6],we introduced a new lattice structure called closed-label concept lattice. In the new structure, all concepts are depicted with its intension reduction. In the following, we briefly introduce some concepts of closed-label concept lattice.

Let (U, A, I) be a formal context, $Y \subseteq A$, then Y is a closed itemset iff $Y'^* = Y$. closed itemset is the maximal set of attributes that shared by some object set. Thus, (1) Every intension of concept in concept lattice is a closed itemset. (2) The set of intension for all concepts in concept lattice equals to the set of closed itemset generated from formal context.

Definition 3. Let (U, A, I) be a formal context. The closed-label of C , expressed as $Closedlabel(C)$. For $\forall Y \in Closedlabel(C)$, satisfied:

(1) $Y'^* \neq Y$

(2) $Y'^* = intension(C)$

(3) $\forall Z \subset Y, Z'^* \subset Y'^*$

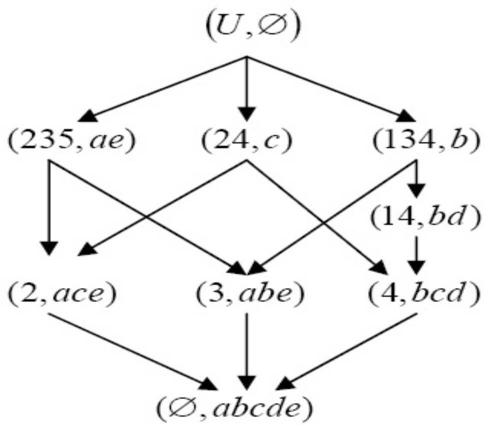

Fig. 1. $L(U, \{a, b, c, d, e\}, I)$

Definition 4. Let (U, A, I) be a formal context. C called a closed-label concept, expressed as $(extension(C), Closedlabel(C), intension(C))$.

The set of all closed-label concepts forms a closed-label concept lattice and is denoted by $CL(U, A, I)$.

Example 2. The closed-label concept lattice of Table 1 is shown in Fig. 2.

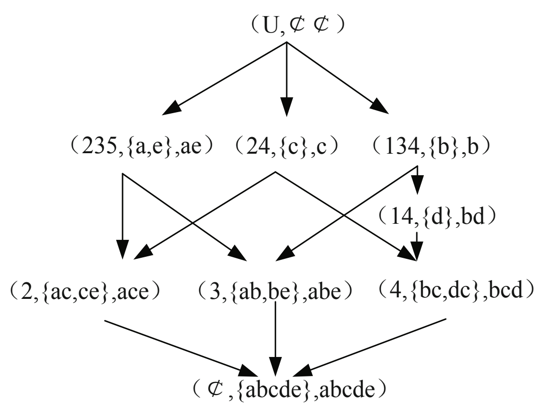

Fig. 2. $CL(U, \{a, b, c, d, e\}, I)$

4 Closed-Label Concept Lattice Based Rule Extraction Approach

In this section, a classification rule extration method based on closed-label concept lattice is given.

Theorem 1. Let $K = (U, A, I)$ be a formal context and $L(U, A, I)$ the associated concept lattice. $L_U(U, A, I)$ is the set of extension of all concepts: $L_U(U, A, I) = \{X | (X, B) \in L(U, A, I)\}$. Then, $L_U(U, A, I) = U$. $L_U(U, A, I)$ forms a cover of the domain.

Proof. The conclusion can be obtained directly.

Let $K^{'} = (U, A \bigcup d, I)$ be a decision table, A the set of condition attributes and d the decision attribute. $\pi_d = \{d_i | i = 1, 2, \cdots, n\}$ denote that d divided U into d class.

Definition 5. Let $K^{'} = (U, A \bigcup d, I)$ be a decision table, $L(U, A, I)$ the associated concept lattice induced by (U, A, I), if $\bigcup\{extention(C) | extension(C) \subseteq d_i, extension(C) \in L_U(U, A, I), i = 1, 2, \cdots, n\} = U$, then $K^{'} = (U, A \bigcup d, I)$ is consistent.

In the following, a classification rule mining method based on closed-label concept lattice is proposed.

Algorithm 1: A classification rule mining method based on closed-label concept lattice

Input: A decision table $K^{'} = (U, A \bigcup d, I)$, where $U = \{u_1, u_2, \cdots, u_{|U|}\}$, $A = \{a_1, a_2, \cdots, a_{|A|}\}$, and associated closed-label concept lattice $CL(U, A, I)$,$\pi_d = \{d_i | i = 1, 2, \cdots, n\}$

Output: A classification rule set R

Initialize $R = \emptyset$,$\Omega = \emptyset$
  For every decision class d_i begin
    Sort the concept as $|extention(C)|$ descending
     for every $extention(C) \in L_U(U, A, I)$
      if $extention(C) \subseteq d_i$, and $extention(C) \nsubseteq \Omega$
      then $R = R \bigcup \{Closedlable(C) \Longrightarrow d_i\}$,$\Omega = \Omega \bigcup extention(C)$
     End
  End
Output R
End

As follows, we analyze the complexity for Algorithm 1. We suppose $|C|$ is the number of concepts in closed label lattice, $|d|$ is the number of decision class. Then, without considering the complexity of lattice construction. The time complexity of this algorithm is $O(|C||d|$.

Example 3. Table 2 depicts an example of decision table $K^{'} = (U, A \bigcup class, I)$, where $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{Height, Hair, Eyes\}$. The corresponding formal context is shown as Table 3. where a denote $Height = short$, b denote $Height = tall$, c denote $Hair = blond$, d denote $Hair = red$, e denote $Hair = dark$, f denote $Eyes = brown$, g denote $Eyes = blue$. Fig. 3 is the Hasse diagram of the closed-label concept lattice derived from Table 2.

Then, four classification rules can be induced based on Algorithm 1: $f \Longrightarrow class = -$, $e \Longrightarrow class = -$, $cg \Longrightarrow class = +$, $d \Longrightarrow class = +$, i.e.,

Table 2. Decision table

	Height	Hair	Eyes	Class
1	short	blond	blue	+
2	short	blond	brown	-
3	tall	red	blue	+
4	tall	dark	blue	-
5	tall	dark	blue	-
6	tall	blond	blue	+
7	tall	dark	brown	-
8	short	blond	brown	-

Table 3. Formal context of Table 2

U	a	b	c	d	e	f	g
1	1	0	1	0	0	0	1
2	1	0	1	0	0	1	0
3	0	1	0	1	0	0	1
4	0	1	0	0	1	0	1
5	0	1	0	0	1	0	1
6	0	1	1	0	0	0	1
7	0	1	0	0	1	1	0
8	1	0	1	0	0	1	0

$r_1 : Eyes = brown \Longrightarrow class = -;$
$r_2 : Hair = dark \Longrightarrow class = -;$
$r_3 : Hair = blond \wedge Eyes = blue \Longrightarrow class = +;$
$r_4 : Hair = red \Longrightarrow class = +.$

5 Comparison with ID3 Algorithm

Decision tree is an instance-based inductive learning algorithm, which induce classification rules from a group of objects. ID3 algorithm [19]proposed by Quinlan is a class of decision tree algorithm, which use information gain as the selection criteria of attribute.

Fig. 4 is the decision tree derived from Table 2 using ID3 Algorithm.

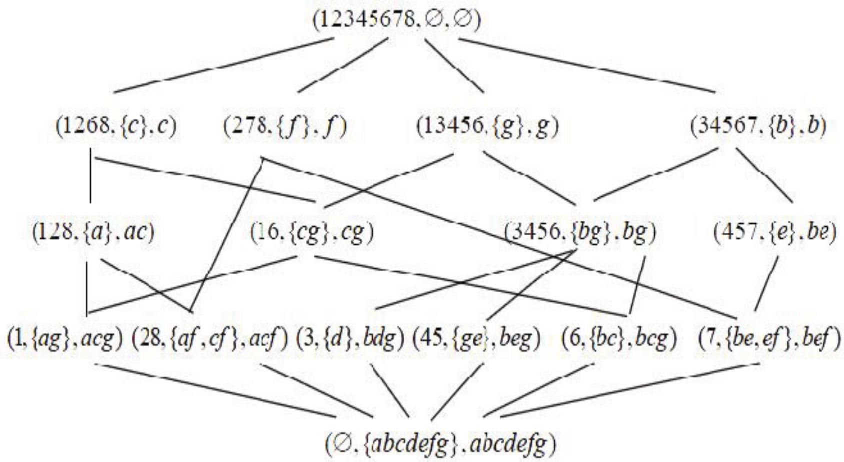

Fig. 3. Hasse diagram

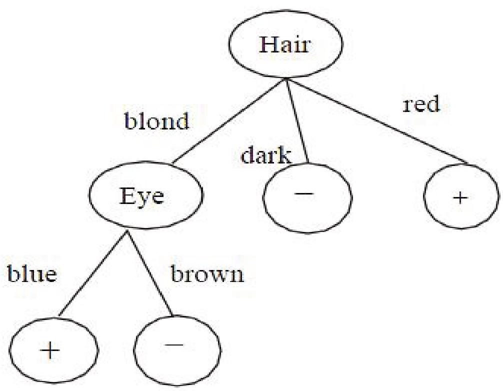

Fig. 4. Decision tree

Then, four classification rules can be induced from Fig. 4:
$r_1^{'}: Eyes = brown \wedge Eyes = brown \Longrightarrow class = -;$
$r_2^{'}: Hair = dark \Longrightarrow class = -;$
$r_3^{'}: Hair = blond \wedge Eyes = blue \Longrightarrow class = +;$
$r_4^{'}: Hair = red \Longrightarrow class = +;$

From the two groups of rules, we can see that r_2, r_3, r_4, and $r_2^{'}, r_3^{'}, r_4^{'}$ has same form, but r_1 has more concise form than $r_1^{'}$.

Also, Hasse diagram provides a description of the whole information table, and can give users a more intuitive choice. In addition, when the test set increasing, concept lattice can be dynamically generated, which avoid the re-construction.

6 Conclusions

Classification is one of typical machine learning issues. As a novel soft computation method, the concept lattice show some advantages for data analysis and extracting rules. This paper has presented a method for classification rules extraction based on one generalization of concept lattice, called closed-label concept lattice. The rules extracted by the proposed method have much simpler forms.

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