Information Granularity and Granular Structure in Decision Making

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Abstract. Multiple criteria decision making (MCDM) has received increasing attentions in both engineering and economic fields. Weights of the criteria directly affect decision results in MCDM, so it is important for us to acquire the appropriate weights of the criteria. In some decision making problems, experts always express their preference by multiplicative preference relation and fuzzy preference relation. In this paper, an objective method based on information granularity is proposed for acquiring weights of the criteria in MCDM. Moreover, we prove that the essence of a consistence preference relation is a partial relation, and analyze the corresponding partial granular structure of the alternative set according to the given partial relation.

Keywords: Information granularity, Granular structure, Multiplicative preference relation, Fuzzy preference relation, Partial relation.

1 Introduction

Decision making is a key issue of the decision theory. One of the most important decision making problems is the multi-criteria decision making problem (MCDM), which is characterized by the ranking of objects according to a set of criteria with pre-defined preference-ordered decision classes. It is widely used in credit approval, stock risk estimation, university ranking, etc[1–5]. In multi-criteria decision making problems, different weighting systems decide different results, so it is important for us to search for a rational weighting method. And weighting methods are classified into subjective method, objective method and the integrated method. Analytic hierarchy Process (AHP) introduced by Satty[6] is a very important approach to support the decision making. Using AHP, the decision maker(s) must compare all pairs of criteria and decision alternatives using a ratio scale to form some judgment matrixes, which are indeed the multiplicative preference relations. Fuzzy preference relation, the other common used

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preference format proposed by Basu[7], is also widely used in group decision making problems[8, 9].

Granular computing is an emerging field of study on human-centered, knowledge intensive problem solving using multiple levels of granularity [10, 11]. Granule, granulation and granularity are regarded as the three primitive notions of granular computing. A granule is a clump of objects drawn together by indistinguishability, similarity, and proximity of functionality [12, 13]. And granulation of an object leads to a collection of granules. The granularity is the measurement of the granulation degree of objects[14]. A triarchic theory of granular computing is proposed by Yao based on the three perspectives on philosophy, methodology and information processing paradigm. In methodological perspective, the granular computing is a structured problem solving method [15]. Of course, the granular computing can be used in the decision making problems, and many researchers have paid their attention to this field [16–19].

Granular computing has been used in decision problem in many fields. Granular reciprocal matrix was proposed by Pedrycz and Song in group decision making problems, and the flexibility offered by the level of granularity is used to increase the level of consensus within the group [16]. Different decision makers may provide multi-granular linguistic information in multi-criteria group decision making problems, so Herrera-Viedma et al. defined the measurements of consensus to help gain the more rational decision results[17], and paper[18] provided a way to use multi-granular linguistic model for management decision making in performance appraisal. In another study, Zheng et al. used granule sets to develop the bi-level decision models[19].

In this paper, we propose a special objective weights based on information granularity in multi-criteria decision problems. Moreover, we analyze the implied preference structure in the two preference relations: multiplicative preference relation and fuzzy preference relation. We prove that a consistent preference relation is indeed a partial relation under the given condition. And the corresponding partial relation induces a partial granular structure. This paper is organized as follows. Section 2 presents an objective weights acquisition method based on information granularity. Section 3 concludes that a consistent preference relation is truly a partial relation, and it can induce a partial granular structure. Section 4 concludes the paper and discusses the future research.

2 Weights Acquisition Based on Information Granularity

Multi-criteria decision making problems(MCDM)could be described by means of the following sets: $U = \{u_1, u_2, \cdots, u_n\} (n \geq 2)$ be a discrete set of n feasible alternatives; $A = \{a_1, a_2, \cdots, a_m\}$ be a finite set of attributes, and $a_i(u_j) = v_{ij}$ denote the value of u_j in the i^{th} attribute; $\omega = (\omega_1, \omega_2, \cdots, \omega_m)$ be a weight vector of attributes, where $\omega_i \geq 0 (i = 1, 2, \cdots, m)$, $\sum_{i=1}^{m} \omega_i = 1$.

Several weighting methods have been summarized in reference[21]. In the following, we give a new weights acquisition method based on information granularity and we suppose all the attributes of the discrete numeric or linguistic values.

Given $a_i \in A$, let $ind(a_i) = \{(u_j, u_k) | a_i(u_j) = a_i(u_k)\}$, apparently, $ind(a_i)$ is the equivalence relation on U, and $U/ind(a_i)$ is the partition of the alternative set U, shortly and conveniently denoted by $U/a_i = \{[u_1]_{a_i}, [u_2]_{a_i}, \cdots, [u_n]_{a_i}\}$.

Definition 1. [20] *Let $U/a_i = \{[u_1]_{a_i}, [u_2]_{a_i}, \cdots, [u_n]_{a_i}\}$, then the information granularity $GR(a_i)$ of the attribute a_i is defined as*

$$GR(a_i) = \frac{1}{n} \sum_{j=1}^{n} \frac{|[u_j]_{a_i}|}{n}. \tag{1}$$

Where $|[u_j]_{a_i}|$ is the cardinality of the equivalence class of u_j .

Property 1. $\frac{1}{n} \leq GR(a_i) \leq 1$

Proof. $1 \leq |[u_j]_{a_i}| \leq n$, so $\frac{1}{n} \leq \frac{1}{n} \sum\limits_{j=1}^{n} \frac{|[u_j]_{a_i}|}{n} \leq 1$.

There are two special cases, the first case is that every alternative has all the same value under the attribute a_i , the other case is that each alternative has unique value under the attribute a_i . In the former case, the attribute a_i contribute nothing to the decision making process, so we can set less weight even 0 for it. While in the other case, the a_i can distinguish the alternatives from each other, so we can set more weight for it. Just like information entropy, the information granulation $GR(a_i)$ can depict the distinguish ability of the attribute a_i . So we propose the weighting method based on information granularity in the following.

Definition 2. *In the MCDM problem given above, the distinguish importance of the attribute a_i is defined as*

$$DI(a_i) = 1 - GR(a_i). \tag{2}$$

Definition 3. *In the MCDM problem given above, the weight of the attribute a_i is defined as*

$$\omega_i = \frac{DI(a_i)}{\sum_{j=1}^{m} DI(a_j)}. \tag{3}$$

Apparently, $\omega_i \geq 0 (i = 1, 2, \cdots, m), \sum_{i=1}^{m} \omega_i = 1$.

Example 1. An evaluation information system is given in table 1, we compute the weights of the criteria by the method given in this section.

$U/a_1 = \{\{u_1, u_3, u_4, u_8, u_9\}, \{u_2, u_7\}, \{u_5, u_6, u_{10}\}\}$

$U/a_2 = \{\{u_1, u_2, u_3, u_4\}, \{u_5, u_6, u_7, u_8, u_9\}, \{u_{10}\}\}$

$U/a_3 = \{\{u_1, u_8\}, \{u_2, u_5, u_{10}\}, \{u_3, u_6\}, \{u_4, u_7, u_9\}\}$

$U/a_4 = \{\{u_1\}, \{u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}\}$

$GR(a_1) = \frac{1}{10} \times (\frac{5}{10} + \frac{2}{10} + \frac{5}{10} + \frac{5}{10} + \frac{3}{10} + \frac{3}{10} + \frac{2}{10} + \frac{5}{10} + \frac{5}{10} + \frac{3}{10}) = \frac{38}{100}$

$GR(a_2) = \frac{1}{10} \times (\frac{4}{10} + \frac{4}{10} + \frac{4}{10} + \frac{4}{10} + \frac{5}{10} + \frac{5}{10} + \frac{5}{10} + \frac{5}{10} + \frac{5}{10} + \frac{1}{10}) = \frac{42}{100}$

$GR(a_3) = \frac{1}{10} \times (\frac{2}{10} + \frac{3}{10} + \frac{2}{10} + \frac{3}{10} + \frac{3}{10} + \frac{2}{10} + \frac{3}{10} + \frac{2}{10} + \frac{3}{10} + \frac{3}{10}) = \frac{26}{100}$

$GR(a_4) = \frac{1}{10} \times (\frac{1}{10} + \frac{9}{10} + \frac{9}{10} + \frac{9}{10} + \frac{9}{10} + \frac{9}{10} + \frac{9}{10} + \frac{9}{10} + \frac{9}{10} + \frac{9}{10}) = \frac{82}{100}$

$$DI(a_1) = 1 - GR(a_1) = 1 - \frac{38}{100} = \frac{62}{100}$$
$$DI(a_2) = 1 - GR(a_2) = 1 - \frac{42}{100} = \frac{58}{100}$$
$$DI(a_3) = 1 - GR(a_3) = 1 - \frac{26}{100} = \frac{74}{100}$$
$$DI(a_4) = 1 - GR(a_3) = 1 - \frac{82}{100} = \frac{18}{100}$$
$$\omega_1 = \frac{DI(a_1)}{\sum_{j=1}^{4} DI(a_j)} = \frac{\frac{62}{100}}{\frac{62}{100}+\frac{58}{100}+\frac{74}{100}+\frac{18}{100}} = 0.292$$
$$\omega_2 = 0.274, \omega_3 = 0.349, \omega_4 = 0.085$$

From the Table 1, we can see that the discernibility of a_3 is the largest in the criteria and ω_3 is the same. The values of the alternatives under a_4 are almost all the same, so the importance of a_4 is the least. We just present the method of weights acquisition of this problem, the aggregation of it is not talked about here.

Table 1. An evaluation information system in a MCDM problem

U	a_1	a_2	a_3	a_4
u_1	good	medium	good	good
u_2	poor	medium	very poor	medium
u_3	good	medium	poor	medium
u_4	good	medium	medium	medium
u_5	medium	poor	very poor	medium
u_6	medium	poor	poor	medium
u_7	poor	poor	medium	medium
u_8	good	poor	good	medium
u_9	good	poor	medium	medium
u_{10}	medium	good	very poor	medium

The weighting method given above is an objective method, it can be used in MCDM, when it is hard to get the subjective weights. And it can also be combined with the subjective weights in integrated methods. Sometimes, the weights determined by objective methods are inconsistent with the DM's subjective preferences. Contrariwise, the judgments of the decision makers occasionally absolutely depend on their knowledge or experience, and the error in weights to some extent is unavoidable. As we all know, none of the two approaches is perfect, and a integrated method might be the most appropriate for determining the weights of criteria [21].

3 The Partial Granular Structure in Preference Relation

3.1 Multiplicative Preference Relation and Fuzzy Preference Relation

In some group decision making problems, decision makers expressed their preference by means of preference relation defined over a finite and fixed set of alternatives. Let $U = \{u_1, u_2, \cdots, u_n\}(n \geq 2)$ be a set of the alternatives. In a preference relation a decision maker associates to every pair of alternatives a value that reflects some degree of preference of the first alternative over the second one. Many important decision models have been developed using main two kinds of preference relations: multiplicative preference relation and fuzzy preference relation.

Multiplicative Preference Relations[22, 23]: A multiplicative preference relation A on the alternative set U is represented by a matrix $A = (a_{ij})$, while a_{ij} is interpreted as u_i is a_{ij} times as good as u_j. Satty suggests measuring a_{ij} using a ratio scale, and precisely the 1-9 scale [6]: $a_{ij} = 1$ indicates indifference between u_i and u_j , $a_{ij} = 9$ indicates u_i is absolutely preference to u_j , and $a_{ij} \in \{2, 3, \cdots, 8\}$ indicates intermediate preference evaluations, while $a_{ij} \in \{\frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{8}\}$ indicates the u_j is intermediate preference to u_i . In this case, the preference A is usually assumed multiplicative reciprocal, i.e., $a_{ij} \cdot a_{ji} = 1, \ \forall i, j \in \{1, 2, \cdots, n\}$.

Definition 4. *A reciprocal multiplicative preference relation $A = (a_{ij})$ is consistent if*

$$a_{ij} \cdot a_{jk} = a_{ik}, \forall i, j, k = 1, 2, \cdots, n. \tag{4}$$

Fuzzy Preference Relations[23]: A fuzzy preference relation B on the alternative set U is a fuzzy set on the product set $U \times U$, that is characterized by a membership function $\mu_B : U \times U \to [0, 1]$.

When cardinality of U is small, the preference relation may be conveniently represented by the $n \times n$ matrix $B = (b_{ij})$ being $b_{ij} = \mu_B(u_i, u_j) \forall i, j = 1, 2, \cdots, n$. b_{ij} is interpreted as the preference degree of the alternative u_i over u_j : $b_{ij} = \frac{1}{2}$ indicates indifference between u_i and $u_j (u_i \sim u_j)$, $b_{ij} = 1$ indicates u_i is absolutely preference to u_j , and $b_{ij} > \frac{1}{2}$ indicates u_i is preference to $u_j (u_i \succ u_j)$. In this case, the preference matrix B is usually assumed addictive reciprocal, i.e.,$b_{ij} + b_{ji} = 1, \ \forall i, j \in 1, 2, \cdots, n$.

Definition 5. [24] *A fuzzy preference relation $B = (b_{ij})$ is consistent if the relation satisfy the additive transitivity condition: reciprocal multiplicative preference relation $A = (a_{ij})$ is consistent if*

$$b_{ij} + b_{jk} - b_{ik} = \frac{1}{2} \ \ or \ \ b_{ij} + b_{jk} + b_{ki} = \frac{3}{2} (\forall i, j, k \in \{1, 2, \cdots, n\}). \tag{5}$$

3.2 Partial Granular Structure

Partial Relation and Consistent Multiplicative Preference Relation

Definition 6. *Let $A = (a_{ij})_{n \times n}$ be a consistent multiplicative preference relation, $\forall \alpha \in [\frac{1}{9}, 9]$, the binary relation R^{A^α} on U is defined as*

$$R^{A^\alpha} = \{(u_i, u_j) | a_{ij} > \alpha \ \ or \ \ i = j\}. \tag{6}$$

Theorem 1. *If $\alpha \geq 1$, the binary relation R^{A^α} induced by the consistent multiplicative relation A is a partial relation.*

Proof. Reflexivity. According to the definition of R^{A^α}, $\forall \mu_i \in U$, $(u_i, u_i) \in R^{A^\alpha}$.

Anti-symmetry. If$(u_i, u_j) \in R^{A^\alpha}$ and $(u_j, u_i) \in R^{A^\alpha}$ hold at the same time, according to the definition of R^{A^α}, and the condition $\alpha \geq 1$, we have the compound proposition $(a_{ij} > 1)$ *or* $(i = j)$ *and* $(a_{ji} > 1)$ *or* $(i = j)$ is true. If $a_{ij} > 1$ is true, then $a_{ji} = 1/a_{ij} < 1$, for the$(a_{ij} > 1)$ *or* $(i = j)$ is true, so $i = j$; If $a_{ij} \leq 1$, for $(a_{ij} > 1)$ *or* $(i = j)$ is true, then $i = j$;. So we can conclude that if $(u_i, u_j) \in R^{A^\alpha}$ and $(u_j, u_i) \in R^{A^\alpha}$ then $u_i = u_j$.

Transitivity. $(u_i, u_j) \in R^{A^\alpha}$ and $(u_j, u_k) \in R^{A^\alpha}$. When $i = j$ or $j = k$, obviously, $(u_i, u_k) \in R^{A^\alpha}$ is true. When $i \neq j$ *and* $j \neq k$, according to the definition of R^{A^α} and the condition $\alpha \geq 1$, we have $a_{ij} > 1, a_{jk} > 1$, then $a_{ik} = a_{ij} \times a_{jk} > 1$, so $(u_i, u_k) \in R^{A^\alpha}$.

This completes the proof.

Partial Relation and Consistent Fuzzy Preference Relation

Definition 7. *Let $B = (b_{ij})_{n\times n}$ be a consistence fuzzy preference relation, $\forall \alpha \in [0, 1]$, the binary relation R^{B^α} on U is defined as*

$$R^{B^\alpha} = \{(u_i, u_j)|b_{ij} > \alpha \;\; or \;\; i = j\}. \tag{7}$$

Theorem 2. *If $\alpha \geq 0.5$, the binary relation R^{B^α} induced by the consistent fuzzy preference relation B is a partial relation.*

Proof. Reflexivity. According to the definition of R^{B^α}, $\forall \mu_i \in U$, $(u_i, u_i) \in R^{B^\alpha}$.

Anti-symmetry. If$(u_i, u_j) \in R^{B^\alpha}$ and $(u_j, u_i) \in R^{B^\alpha}$ hold at the same time, according to the definition of R^{B^α}, and the condition $\alpha \geq 0.5$, we have the compound proposition $(b_{ij} > 0.5)$ *or* $(i = j)$ *and* $(b_{ji} > 0.5)$ *or* $(i = j)$ is true. If $b_{ij} > 0.5$ is true, then $b_{ji} = 1 - b_{ij} < 0.5$, for the$(b_{ij} > 0.5)$ *or* $(i = j)$ is true, so $i = j$; If $b_{ij} \leq 0.5$, for $(b_{ij} > 0.5)$ *or* $(i = j)$ is true, then $i = j$;. So we can conclude that if $(u_i, u_j) \in R^{B^\alpha}$ and $(u_j, u_i) \in R^{B^\alpha}$ then $u_i = u_j$.

Transitivity. $(u_i, u_j) \in R^{B^\alpha}$ and $(u_j, u_k) \in R^{B^\alpha}$. When $i = j$ or $j = k$, obviously, $(u_i, u_k) \in R^{B^\alpha}$ is true. When $i \neq j$ *andj* $\neq k$, according to the definition of R^{B^α} and the condition $\alpha \geq 0.5$, we have $b_{ij} > 0.5, b_{jk} > 0.5$, then $b_{ik} = b_{ij} + b_{jk} - 0.5 > 0.5$, so $(u_i, u_k) \in R^{B^\alpha}$.

This completes the proof.

Let $U = \{u_1, u_2, \cdots, u_n\}$ be a set of alternatives, and P be a partial relation on U, $(u_i, u_j) \in P$, shortly, denoted by $u_i \succeq_P u_j$, means u_i preference to u_j under the partial relation P. The granule of knowledge induced by partial relation P is the set of objects dominating u_i, i.e. $[u_i]^{\succeq_P} = \{u_j | u_j \succeq u_i\}$.

Definition 8. *Let $U = \{u_1, u_2, \cdots, u_n\}$ be a set of alternatives, and P be a partial relation on U, define $G_P(U)$ is the partial granular structure of U induced by P, and denoted as*

$$G_P(U) = \{[u_1]^{\succeq_P}, [u_2]^{\succeq_P}, \cdots, [u_n]^{\succeq_P}\} \tag{8}$$

An approach to sorting for objects in set-valued ordered information systems are given based on the partial granulation induced by a partial relation in [25]. In this paper, we only discuss the partial granular structure induced by the two kinds of preference relation, and the applications are the further researches. The example given in the following is to demonstrate the partial granular structure implied in the consistent multiplicative preference relation and fuzzy preference relation.

Example 2. Suppose that we have a set of four alternatives $\{u_1, u_2, u_3, u_4, u_5\}$ and a decision maker gives his or her consistent multiplicative preference relation as follows:

$$A = \begin{bmatrix} 1 & \frac{1}{2} & 3 & 4 & 1 \\ 2 & 1 & 6 & 8 & 2 \\ \frac{1}{3} & \frac{1}{6} & 1 & \frac{4}{3} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{8} & \frac{3}{4} & 1 & \frac{1}{4} \\ 1 & \frac{1}{2} & 3 & 4 & 1 \end{bmatrix},$$

Then, we get

$R^{A^1} = \{(u_1, u_1), (u_1, u_3), (u_1, u_4), (u_2, u_1), (u_2, u_2), (u_2, u_3), (u_2, u_4), (u_2, u_5), (u_3, u_3), (u_3, u_4), (u_4, u_4), (u_5, u_3), (u_5, u_4), (u_5, u_5)\}$

$R^{A^3} = \{(u_1, u_1), (u_1, u_4), (u_2, u_2), (u_2, u_3), (u_2, u_4), (u_3, u_3), (u_4, u_4), (u_5, u_4), (u_5, u_5)\}$

$R^{A^6} = \{(u_1, u_1), (u_2, u_2), (u_2, u_4), (u_3, u_3), (u_4, u_4), (u_5, u_5)\}$

It is easily to prove that R^{A^1},R^{A^3},R^{A^8} are partial relations, and their corresponding partial granular structure are:

$$\begin{aligned} G_{R^{A^1}}(U) &= \{[u_1]^{\succeq_{R^{A^1}}}, [u_2]^{\succeq_{R^{A^1}}}, [u_3]^{\succeq_{R^{A^1}}}, [u_4]^{\succeq_{R^{A^1}}}, [u_5]^{\succeq_{R^{A^1}}}\} \\ &= \{\{u_2, u_1\}, \{u_2\}, \{u_1, u_2, u_3, u_5\}, \{u_1, u_2, u_3, u_4, u_5\}, \{u_2, u_5\}\}, \end{aligned}$$

$$\begin{aligned} G_{R^{A^3}}(U) &= \{[u_1]^{\succeq_{R^{A^3}}}, [u_2]^{\succeq_{R^{A^3}}}, [u_3]^{\succeq_{R^{A^3}}}, [u_4]^{\succeq_{R^{A^3}}}, [u_5]^{\succeq_{R^{A^3}}}\} \\ &= \{\{u_1\}, \{u_2\}, \{u_2, u_3\}, \{u_1, u_2, u_4, u_5\}, \{u_5\}\}, \end{aligned}$$

$$\begin{aligned} G_{R^{A^6}}(U) &= \{[u_1]^{\succeq_{R^{A^6}}}, [u_2]^{\succeq_{R^{A^6}}}, [u_3]^{\succeq_{R^{A^6}}}, [u_4]^{\succeq_{R^{A^6}}}, [u_5]^{\succeq_{R^{A^6}}}\} \\ &= \{\{u_1\}, \{u_2\}, \{u_3\}, \{u_2, u_4\}, \{u_5\}\}. \end{aligned}$$

From the example, we can see the smaller value of $\alpha \geq 1$, the more preference information can draw from the corresponding granular structure. So we can get the finest preference order is:

$$u_2 \succ \begin{pmatrix} u_1 \\ u_5 \end{pmatrix} \succ u_3 \succ u_4.$$

Example 3. Suppose that we have a set of four alternatives $\{u_1, u_2, u_3, u_4\}$ and a decision maker gives his or her fuzzy preference relation as follows:

$$B = \begin{bmatrix} 0.5 & 0.55 & 0.7 & 0.95 \\ 0.45 & 0.5 & 0.65 & 0.9 \\ 0.3 & 0.35 & 0.5 & 0.75 \\ 0.05 & 0.1 & 0.25 & 0.5 \end{bmatrix},$$

It is easy to prove that B is a consistent fuzzy preference relation. According to the definition 7, we get

$$R^{B^{0.5}} = \{(u_1, u_1), (u_2, u_2), (u_3, u_3), (u_4, u_4), (u_1, u_2), (u_1, u_3), (u_1, u_4), (u_2, u_3), (u_2, u_4), (u_3, u_4)\},$$

$$R^{B^{0.7}} = \{(u_1, u_1), (u_2, u_2), (u_3, u_3), (u_4, u_4), (u_1, u_4)(u_2, u_4), (u_3, u_4), \},$$

$$R^{B^{0.95}} = \{(u_1, u_1), (u_2, u_2), (u_3, u_3), (u_4, u_4)\}.$$

And it is easy to see that the three relations $R^{B^{0.5}}$,$R^{B^{0.7}}$,$R^{B^{0.95}}$ are all partial relations, their respective partial granular structure are as follows:

$$G_{R^{B^{0.5}}}(U) = \{[u_1]^{\succeq}_{R^{B^{0.5}}}, [u_2]^{\succeq}_{R^{B^{0.5}}}, [u_3]^{\succeq}_{R^{B^{0.5}}}, [u_4]^{\succeq}_{R^{B^{0.5}}}\} = \{\{u_1\}, \{u_1, u_2\}, \{u_1, u_2, u_3\}, \{u_1, u_2, u_3, u_4\}\},$$

$$G_{R^{B^{0.7}}}(U) = \{[u_1]^{\succeq}_{R^{B^{0.7}}}, [u_2]^{\succeq}_{R^{B^{0.7}}}, [u_3]^{\succeq}_{R^{B^{0.7}}}, [u_4]^{\succeq}_{R^{B^{0.7}}}\} = \{\{u_1\}, \{u_2\}, \{u_3\}, \{u_1, u_2, u_3, u_4\}\},$$

$$G_{R^{B^{0.95}}}(U) = \{[u_1]^{\succeq}_{R^{B^{0.95}}}, [u_2]^{\succeq}_{R^{B^{0.95}}}, [u_3]^{\succeq}_{R^{B^{0.95}}}, [u_4]^{\succeq}_{R^{B^{0.95}}}\} = \{\{u_1\}, \{u_2\}, \{u_3\}, \{u_4\}\}.$$

From this example, we can see the smaller value of $\alpha \geq 0.5$, the more preference information can draw from the corresponding granular structure. So we can get the finest preference order is:

$$u_1 \succ u_2 \succ u_3 \succ u_4.$$

4 Conclusions

To recapitulate, weight acquiring method is very important in multiple criteria decision making, and constructing granular structure of the set of alternatives is also helpful to comprehend the judgment of a decision maker. In this paper, we have proposed a weights acquisition method based on information granularity, and it can be combined with subjective weighting method to decide the final weights of the criteria. Moreover, we have proved that the partial structure is implied in the two preference relations. And a partial granular structure can be induced by the special partial relation implied in the two preference relation. The interesting topics for further study would be to construct suitable granules to solve decision making problems.

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