ON PARTIAL ORDER RELATIONS IN GRANULAR COMPUTING

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Abstract

Information granularity is an approache to measuring uncertainty of an information system in granular computing. Partial order relations are often used to characterize the monotonicity of an uncertainty measure. In this paper, we focus on a generalized partial relation $\preceq^{'}$ with set-size character to information systems, and prove that three existing partial relations ($\preceq_1$, $\preceq_2$ and $\preceq_3$) are all its special cases. The analysis shows that the partial relation $\preceq^{'}$ presented appears to be well suited to characterize the essential natures of information granularity in granular computing.

Keywords:

Information granularity; partial order relation; Information system; Granular computing

1. Introduction

The notion of information systems (sometimes called data tables, attribute-value systems, knowledge representation systems, etc.), provides a convenient tool for the representation of objects in terms of their attribute values [2–4,8]. The use of any binary relation results in information granulation of information systems, such as equivalence relation [13], tolerance relation [12] and maximal consistent relation [4]. Measuring the uncertainty of a given information system is a very important and significant issue in granular computing [1].

According to whether or not there are missing data, information systems are classified into two categories: complete and incomplete [5, 10]. Partial relation is always used to research information entropy, information granularity, measuring knowledge content, measuring the significance of an attribute and their applications in information systems. Partial relation $\preceq_1$ has been introduced to research some properties of complete information systems [6, 11]. In incomplete information systems, many researchers usually investigate its some characters through using partial relation $\preceq_2$ [3, 7]. To consider minimal information granules in an incomplete information system, Leung and Li [4] applied the concept of a maximal consistent block to formulate a new approximation. This method has been used for attribute reduction and rule acquisition in an incomplete information system. However, the partial relations $\preceq_1$ and $\preceq_2$ can not be used to characterize the uncertainty of an incomplete information system in the context of maximal consistent blocks. For this objective, Qian et al. [12] introduced a new partial relation $\preceq_3$ to depict the relationship between covers induced by two maximal consistent relations.

However, these three partial relations have not a uniform depiction. To obtain a uniform form, in the literature [5], through using the partial relation $\preceq_2$, Liang and Qian presented an axiomatic definition of information granularity in information systems, in which several existing forms of information granularity become its special cases. Qian et al. [10] argued that the size of information granularity is not depended on the sizes of equivalence classes (tolerance classes and maximal consistent blocks), but some array of these classes. To discover the essential nature of this problem, Qian et al. [10] proposed a generalized partial relation $\preceq^{'}$ for characterizing the monotonicity of information granularity, called granulation monotonicity.

The reset of this paper is organized as follows. In Section 2, we review some basic concepts, such as complete information systems, incomplete information systems and maximal consistent block technique. By analyzing the structure of partial relations $\preceq_1$, $\preceq_2$, $\preceq_3$ and $\preceq^{'}$, in Section 3, we establish the relationships among these partial relations. Finally, Section 6 concludes the whole paper.

2. Preliminaries

Information systems provides a convenient tool for the representation of objects in terms of their attribute values

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[2–4]. In this section, we briefly review three kinds of information systems, which are complete information systems, incomplete information systems and information systems based on maximal consistent block technique.

An information system is a pair $S = (U, A)$, where,

(1) U is a non-empty finite set of objects;

(2) A is a non-empty finite set of attributes;

and

(3) for every $a \in A$, there is a mapping a, $a : U \to V_a$, where V_a is called the value set of a.

For an information system $S = (U, A)$, if $\forall a \in A$, every element in V_a is a definite value, then S is called a complete information system.

Each subset of attributes $P \subseteq A$ determines a binary indistinguishable relation $IND(P)$ as $IND(P) = \{(u, v) \in U \times U \mid \forall a \in P, a(u) = a(v)\}$. It is easily shown that $IND(P) = \bigcup_{a \in P} IND(\{a\})$ [9].

$U/IND(P)$ constitutes a partition of U. $U/IND(P)$ is called a knowledge on U and every equivalence class is called a knowledge granule or information granule. In a broad sense, information granularity denotes average measure of information granules induced by P.

It may happen that some of the attribute values for an object are missing. These missing values can be represented by the set of all possible values for the attribute or equivalent to the domain of the attribute. To indicate such a situation, a distinguished value, a so-called null value is usually assigned to those attributes.

If V_a contains a null value for at least one attribute $a \in A$, then S is called an incomplete information system [3], otherwise it is complete. Further on, we will denote the null value by $*$.

Let $S = (U, A)$ be an information system, $P \subseteq A$ an attribute set. We define a binary relation on U as $SIM(P) = \{(u, v) \in U \times U \mid \forall a \in P, a(u) = a(v) \ \text{ or } \ a(u) = * \ \text{ or } \ a(v) = *\}$.

In fact, $SIM(P)$ is a tolerance relation on U, the concept of a tolerance relation has a wide variety of applications in classification [7]. It can be easily shown that $SIM(P) = \bigcap_{a \in P} SIM(\{a\})$.

Let $U/SIM(P)$ denote the family sets $\{S_P(u) \mid u \in U\}$, the classification induced by P. A member $S_P(u)$ from $U/SIM(P)$ will be called a tolerance class or a granule of information. It should be noticed that the tolerance classes in $U/SIM(P)$ do not constitute a partition of U in general. They constitute a covering of U, i.e., $S_P(u) \neq \emptyset$ for every $u \in U$, and $\bigcup_{u \in U} S_P(u) = U$. Of course, $SIM(P)$ degenerates into an equivalence relation in a complete information system.

However, tolerance classes are not the minimal units for describing knowledge or information in incomplete information systems [4].

Let $S = (U, A)$ be an information system, $P \subseteq A$ an attribute set and $X \subseteq U$ a subset of objects. We say X is consistent with respect to P if $(u, v) \in SIM(P)$ for any $u, v \in X$. If there does not exist a subset $Y \subseteq U$ such that $X \subset Y$, and Y is consistent with respect to P, then X is called a maximal consistent block of P. Obviously, in a maximal consistent block, all objects are not indiscernible with available information provided by a similarity relation [4].

Henceforth, we denote the set of all maximal consistent blocks determined by $P \subseteq A$ as C_P, and the set of all maximal consistent blocks of P which includes some object $u \in U$ is denoted as $C_P(u)$. It is obvious that $X \in C_P$ if and only if $X = \bigcap_{u \in X} S_P(u)$ [4].

3. Relationships among partial order relations in granular computing

In this section, we will establish the relationships among the existing partial order relations.

Firstly, we give the definition of each of two partial relations ($\preceq_1$ and $\preceq_2$) and analyze several relative properties. Definition 1 gives the definition of partial relation in complete information systems.

Definition 1 *[6] Let $S = (U, A)$ be a complete information system, $P, Q \subseteq A$. $U/IND(P) = \{P_1, P_2, \cdots, P_m\}$ and $U/IND(Q) = \{Q_1, Q_2, \cdots, Q_n\}$. One defines partial relation $\preceq_1$ as follows*

$P \preceq_1 Q \Leftrightarrow$ for every $P_i \in U/IND(P)$, there exists $Q_j \in U/IND(Q)$ such that $P_i \subseteq Q_j$.

If $P \preceq_1 Q$ and $P \neq Q$, i.e., for some $P_{i_0} \in U/IND(P)$, there exists $Q_{j_0} \in U/IND(Q)$ such that $P_{i_0} \subset Q_{j_0}$, denoted by $P \prec_1 Q$.

In incomplete information systems, the characterization of information are often based on the tolerance classes induced by all objects on the universe. Uncertainty of an incomplete information system is characterized by the partial relation $\preceq_2$, which is shown as follows.

Definition 2 *[7] Let $S = (U, A)$ be an incomplete information system, $P, Q \subseteq A$, $U/SIM(P) = \{S_P(u_1), S_P(u_2), \cdots, S_P(u_{|U|})\}$ and $U/SIM(Q) = \{S_Q(u_1), S_Q(u_2), \cdots, S_Q(u_{|U|})\}$. One defines partial relation $\preceq_2$ as follows*

$$P \preceq_2 Q \Leftrightarrow S_P(u_i) \subseteq S_Q(u_i), \forall i \in \{1, 2, \cdots, |U|\}.$$

If $P \preceq_2 Q$, we say that Q is coarser than P (or P is finer than Q). If $P \preceq_2 Q$ and $P \neq Q$, we say that Q is strictly coarser than P (or P is strictly finer than Q), denoted by $P \prec_2 Q$. In fact, $P \prec_2 Q \Leftrightarrow$ for $\forall i \in$

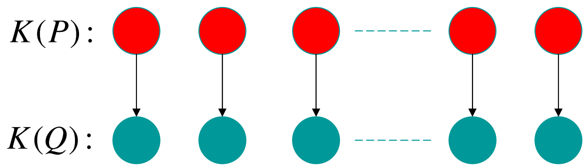

Figure 1: Sketch map of the partial relation $\preceq_2$

$\{1, 2, \cdots, |U|\}$, one has that $S_P(u_i) \subseteq S_Q(u_i)$, and there exists $j \in \{1, 2, \cdots, |U|\}$, such that $S_P(u_j) \subset S_Q(u_j)$.

The Figure 1 gives the sketch map of the partial relation $\preceq_2$. In Figure 1, $K(P) = U/SIM(P) = \{S_P(u_1), S_P(u_2), \cdots, S_P(u_{|U|})\}$ and $K(Q) = U/SIM(P) = \{S_Q(u_1), S_Q(u_2), \cdots, S_Q(u_{|U|})\}$. From Figure 1, it is easy to see that for arbitrary $i \leq |U|$, one has that $S_P(u_i) \subseteq S_Q(u_i)$ and $|S_P(u_i)| \leq |S_Q(u_i)|$. In other words, the cardinality of the tolerance class induced by each object from $K(P)$ is less than or equal to that of the corresponding tolerance class induced by it from $K(Q)$.

From Definition 1 and Definition 2, one can obtain the following theorem.

Theorem 1 *Partial relation $\preceq_1$ is a special instance of partial relation $\preceq_2$.*

Proof. Let $S = (U, A)$ be a complete information system, $P, Q \subseteq A$ with $P \preceq_1 Q$, $U/IND(P) = \{P_1, P_2, \cdots, P_m\}$ and $U/IND(Q) = \{Q_1, Q_2, \cdots, Q_n\}$. Suppose that $U/SIM(P) = \{S_P(u_1), S_P(u_2), \cdots, S_P(u_{|U|})\}$, $P_i = \{u_{i1}, u_{i2}, \cdots, u_{is_i}\}$, where $|P_i| = s_i$, $\sum_{i=1}^{m} s_i = |U|$; $U/SIM(Q) = \{S_Q(u_1), S_Q(u_2), \cdots, S_Q(u_{|U|})\}$, $Q_j = \{u_{j1}, u_{j2}, \cdots, u_{js_j}\}$, where $|Q_j| = s_j$, $\sum_{j=1}^{n} s_j = |U|$.

Hence, we have that

$$P_i = S_P(u_{i1}) = S_P(u_{i2}) = \cdots = S_P(u_{is_i}),$$
$$Q_j = S_Q(u_{j1}) = S_Q(u_{j2}) = \cdots = S_Q(u_{js_j}).$$

Since $P \preceq_1 Q$, we obtain that for every $P_i \in U/IND(P)$, there exists $Q_j \in U/IND(Q)$ such that $P_i = S_P(u_{ik}) \subseteq Q_j = S_Q(u_{jt})$ $(k \leq s_i, t \leq s_j)$, i.e., for arbitrary $u \in U$, we have that $S_P(u) \subseteq S_Q(u)$.

Therefore, the partial relation $\preceq_1$ is a special instance of the partial relation $\preceq_2$.

Similar to the definitions of $\preceq_1$ and $\preceq_2$, we define another partial relation in order to discuss maximal consistent block structure in incomplete information systems. It is shown as follows.

Definition 3 *[12] Let $S = (U, A)$ be an incomplete information system, $P, Q \subseteq A$, $C_P = \{P^1, P^2, \cdots, P^m\}$ and $C_Q = \{Q^1, Q^2, \cdots, Q^n\}$. One defines partial relation $\preceq_3$ by*

$P \preceq_3 Q \Leftrightarrow$ for every $P^i \in C_P$, there exists $Q^j \in C_Q$ such that $P^i \subseteq Q^j$.

If $P \preceq_3 Q$ and $P \neq Q$, i.e., for some $P^{i_0} \in C_P$, there exists $Q^{j_0} \in C_Q$ such that $P^{i_0} \subset Q^{j_0}$, denoted by $P \prec_3 Q$.

The following Theorem 2 and Theorem 3 establish the relationship between $\preceq_1$ and $\preceq_3$ and that between $\preceq_2$ and $\preceq_3$, respectively.

Theorem 2 *Partial relation $\preceq_1$ is a special instance of partial relation $\preceq_3$.*

Proof. If $S = (U, A)$ is a complete information system, then the tolerance relation degenerates into the corresponding equivalence relation, and each of maximal tolerance blocks degenerates into the corresponding equivalence class. Therefore, if $P \preceq_1 Q$ $(P, Q \subseteq A)$, then $P \preceq_3 Q$ holds.

Hence, the partial relation $\preceq_1$ is a special instance of the partial relation $\preceq_3$.

Theorem 3 *Partial relation $\preceq_3$ is a special instance of partial relation $\preceq_2$.*

Proof. Let $S = (U, A)$ be an incomplete information system, $P, Q \subseteq A$ with $P \preceq_3 Q$, $C_P = \{P^1, P^2, \cdots, P^m\}$ and $C_Q = \{Q^1, Q^2, \cdots, Q^n\}$. It follows from the definition of $\preceq_3$ that for arbitrary $P^i \in C_P$, there exists $Q^j \in C_Q$ such that $P^i \subseteq Q^j$.

Next, we prove that $S_P(u) \subseteq S_Q(u)$, $\forall u \in U$. Assume that $C_P(u) = \{X_1, X_2, \cdots, X_m\}$ and $C_Q(u) = \{Y_1, Y_2, \cdots, Y_n\}$. We know that $S_P(u) = \bigcup\{X_k \in C_P \mid X_k \subseteq S_P(u)\} = \bigcup\{X_k \in C_P(u)\}$ $(k \leq m)$ and $S_Q(u) = \bigcup\{Y_t \in C_Q \mid Y_t \subseteq S_Q(u)\} = \bigcup\{Y_t \in C_Q(u)\}$ $(t \leq n)$ from property 4 in reference [16]. From the definition of a maximal consistent block, we have that $u \in C_P(u)$, $u \in C_Q(u)$, $u \notin C_P - C_P(u)$ and $u \notin C_Q - C_Q(u)$. Hence, it follows from $P \preceq_3 Q$ that for arbitrary $X_k \in C_P(u)$, there exists $Y_t \in C_Q(u)$ such that $X_k \subseteq Y_t$. Thus, for arbitrary $u \in U$, one has that

$$\begin{aligned} S_P(u) &= \bigcup\{X_k \in C_P \mid X_k \subseteq S_P(u)\} = \bigcup_{k=1}^{m} X_k \\ &\subseteq \bigcup_{t=1}^{n} Y_t = \bigcup\{Y_t \in C_Q \mid Y_t \subseteq S_Q(u)\} \\ &= S_Q(u), \end{aligned}$$

that is $P \preceq_2 Q$.

Hence, the partial relation $\preceq_3$ is a special instance of the partial relation $\preceq_2$.

Definition 4 *Let $S = (U, A)$ be an information system, $P, Q \subseteq A$, $K(P) = \{S_P(u) \mid u \in U\}$ and $K(Q) = \{S_Q(u) \mid u \in U\}$. One defines a binary relation $\preceq'$ with set size character as follows*

$P \preceq' Q \Leftrightarrow$ for $K(P) = \{S_P(u_1), S_P(u_2), \cdots, S_P(u_{|U|})\}$, there exists a sequence $K'(Q)$ of $K(Q)$, where $K'(Q)$

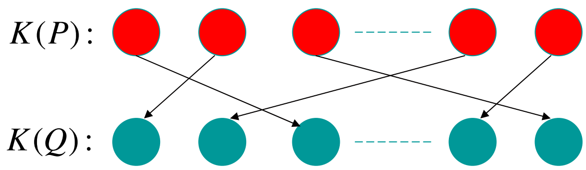

Figure 2: Sketch map of the partial relation $\preceq^{'}$

$= \{S_Q(u_1^{'}), S_Q(u_2^{'}), \cdots, S_Q(u_{|U|}^{'})\}$, *such that* $|S_P(u_i)| \leq |S_Q(u_i^{'})|$.

If there exists a sequence $K^{'}(Q)$ of $K(Q)$ such that $|S_P(u_i)| \leq |S_Q(u_i^{'})|$ and $|S_P(u_{i_0})| < |S_Q(u_{i_0})|$ for some $u_{i_0} \in U$, then we say that P is strictly granularity finer than Q, denoted by $P \prec^{'} Q$; if there exists a sequence $K^{'}(Q)$ of $K(Q)$ such that $|S_P(u_i)| = |S_Q(u_i^{'})|$ for arbitrary $u \in U$, then we say that P is granularity equal to Q, denote by $P \approx Q$.

One can intuitively understand the meaning of this partial relation $\preceq^{'}$ by Figure 2. In Figure 2, $K(P) = U/SIM(P) = \{S_P(u_1), S_P(u_2), \cdots, S_P(u_{|U|})\}$ and $K(Q) = U/SIM(P) = \{S_Q(u_1), S_Q(u_2), \cdots, S_Q(u_{|U|})\}$. In this figure, we use an arrow to denote the size relationship between two tolerance classes. That is to say, if there is an arrow between two tolerance classes, then we can know that the size of one is less than or equal to that of the other. This figure shows that for $K(P) = \{S_P(u_1), S_P(u_2), \cdots, S_P(u_{|U|})\}$, there exists a sequence $K^{'}(Q)$ of $K(Q)$, where $K^{'}(Q) = \{S_Q(u_1^{'}), S_Q(u_2^{'}), \cdots, S_Q(u_{|U|}^{'})\}$, such that $|S_P(u_i)| \leq |S_Q(u_i^{'})|, i \leq |U|$.

Let $S = (U, A)$ be an information system and 2^A the power set induced by A. From the definition of the binary relation $\preceq^{'}$, one can get that $(2^A, \preceq^{'})$ is a partial set and $\preceq^{'}$ on the power set of A is a partial relation.

From the above results, we then establish the relationship between the partial relation $\preceq^{'}$ and each of $\preceq_1$, $\preceq_2$ and $\preceq_3$, which are shown in the following three theorems.

Theorem 4 *Partial relation $\preceq_1$ is a special instance of partial relation $\preceq^{'}$.*

Proof. Let $S = (U, A)$ be a complete information system, $P, Q \subseteq A$ with $P \preceq_1 Q$, $U/IND(P) = \{P_1, P_2, \cdots, P_m\}$ and $U/IND(Q) = \{Q_1, Q_2, \cdots, Q_n\}$. Suppose that $U/SIM(P) = \{S_P(u_1), S_P(u_2), \cdots, S_P(u_{|U|})\}$, $P_i = \{u_{i1}, u_{i2}, \cdots, u_{is_i}\}$, where $|P_i| = s_i$, $\sum_{i=1}^{m} s_i = |U|$; $U/SIM(Q) = \{S_Q(u_1), S_Q(u_2), \cdots, S_Q(u_{|U|})\}$, $Q_j = \{u_{j1}, u_{j2}, \cdots, u_{js_j}\}$, where $|Q_j| = s_j$, $\sum_{j=1}^{n} s_j = |U|$.

Hence, we have that

$$P_i = S_P(u_{i1}) = S_P(u_{i2}) = \cdots = S_P(u_{is_i}),$$
$$Q_j = S_Q(u_{j1}) = S_Q(u_{j2}) = \cdots = S_Q(u_{js_j}).$$

i.e.,

$$|P_i| = |S_P(u_{i1})| = |S_P(u_{i2})| = \cdots = |S_P(u_{is_i})|,$$
$$|Q_j| = |S_Q(u_{j1})| = |S_Q(u_{j2})| = \cdots = |S_Q(u_{js_j})|.$$

From $P \preceq_1 Q$, we obtain that for every $P_i \in U/IND(P)$, there exists $Q_j \in U/IND(Q)$ such that $P_i = S_P(u_{ik}) \subseteq Q_j = S_Q(u_{jt})$ $(k \leq s_i, t \leq s_j)$, i.e., $|P_i| = |S_P(u_{ik})| \leq |Q_j| = |S_Q(u_{jt})|$. That is to say, one can find an array of all tolerance classes in $U/IND(Q)$ such that $P \preceq^{'} Q$.

Therefore, the partial relation $\preceq_1$ is a special instance of partial relation $\preceq^{'}$.

Theorem 5 *Partial relation $\preceq_2$ is a special instance of partial relation $\preceq^{'}$.*

Theorem 6 *Partial relation $\preceq_3$ is a special instance of partial relation $\preceq^{'}$.*

Proof. Let $S = (U, A)$ be an incomplete information system, $P, Q \subseteq A$ with $P \preceq_3 Q$, $C_P = \{P^1, P^2, \cdots, P^m\}$ and $C_Q = \{Q^1, Q^2, \cdots, Q^n\}$. It follows from the definition of $\preceq_3$ that for arbitrary $P^i \in C_P$, there exists $Q^j \in C_Q$ such that $P^i \subseteq Q^j$ and $|P^i| < |Q^j|$.

Next, we prove that $|S_P(u)| \leq |S_Q(u)|, \forall u \in U$. Assume that $C_P(u) = \{X_1, X_2, \cdots, X_m\}$ and $C_Q(u) = \{Y_1, Y_2, \cdots, Y_n\}$. We know that $S_P(u) = \bigcup\{X_k \in C_P \mid X_k \subseteq S_P(u)\} = \bigcup\{X_k \in C_P(u)\}$ $(k \leq m)$ and $S_Q(u) = \bigcup\{Y_t \in C_Q \mid Y_t \subseteq S_Q(u)\} = \bigcup\{Y_t \in C_Q(u)\}$ $(t \leq n)$ from property 4 in the literature [16]. From the definition of maximal consistent block, we have that $u \in C_P(u)$, $u \in C_Q(u)$, $u \notin C_P - C_P(u)$ and $u \notin C_Q - C_Q(u)$. Hence, it follows from $P \preceq_3 Q$ that for arbitrary $X_k \in C_P(u)$, there exist $Y_t \in C_Q(u)$ such that $X_k \subseteq Y_t$. So, $\bigcup_{k=1}^{m} X_k \subseteq \bigcup_{t=1}^{n} Y_t$ and $|\bigcup_{k=1}^{m} X_k| \leq |\bigcup_{t=1}^{n} Y_t|$.

Therefore, we have that

$$\begin{aligned} |S_P(u)| &= |\bigcup\{X_i \in C_P(u)\}| = |\bigcup_{k=1}^{m} X_k| \\ &\leq |\bigcup_{t=1}^{n} Y_t| = |\bigcup\{Y_t \in C_Q(u)\}| \\ &= |S_Q(u)|. \end{aligned}$$

Hence, $|S_P(u)| \leq |S_Q(u)|, \forall u \in U$, i.e., $P \preceq^{'} Q$.

Based on the above analysis, the relationship among the four types of partial relations in information systems can be summarized, and the corresponding superset-subset relationship graph is depicted in Figure 3.

In Figure 3, an arrow stands for a generalization between two partial relations. For example, "$\preceq^{'} \longrightarrow \preceq_1$" means "the partial relation $\preceq^{'}$ is a generalization of the partial relation $\preceq_1$". Thus, in an information system, $\preceq^{'}$ is the most generalized partial relation, and $\preceq_1$ is the most special partial relation. From the viewpoint of generalized ability, one

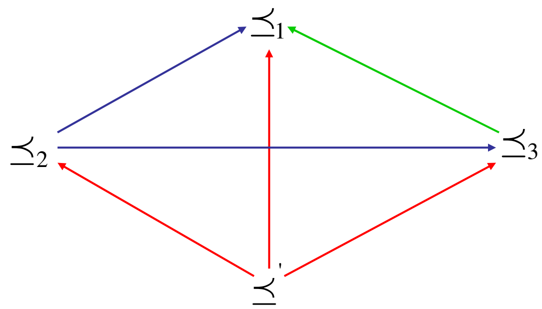

Figure 3: Relationship among the four partial relations

can obtain an array of these partial relations from strong to weak as follows.

$$\preceq' \longrightarrow \preceq_2 \longrightarrow \preceq_3 \longrightarrow \preceq_1 .$$

4. Conclusions

In the present research, a generalized partial relation $\preceq'$ with set size character in information systems has been discussed, and the relationships between $\preceq'$ and three existing partial relations have been established. These relationships show that the partial relation $\preceq'$ may be better to characterize the essential nature of information granularity and information entropy for measuring uncertainty of information in information systems than the partial relations $\preceq_1$, $\preceq_2$ and $\preceq_3$.

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References

[1] I. Düntsch, G. Gediga, Uncertainty measures of rough set prediction, Artificial Intelligence, 106 (1998) 109-137.

[2] J.W. Guan, D.A. Bell, Rough computational methods for information systems, Artificial Intelligence, 105 (1998) 77-103.

[3] M. Kryszkiewicz, Rough set approach to incomplete information systems, Information Sciences 112 (1998) 39-49.

[4] Y. Leung, D.Y. Li, Maximal consistent block technique for rule acquisition in incomplete information systems, Information Sciences 153 (2003) 85-106.

[5] J.Y. Liang, Y.H. Qian, Information granules and entropy theory in information systems, Science in China Series F: Information Sciences, 51(10) (2008) 1427-1444.

[6] J.Y. Liang, Z.Z. Shi, The information entropy, rough entropy and knowledge granulation in rough set theory, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 12(1) (2004) 37-46.

[7] J.Y. Liang, Z.Z. Shi, D.Y. Li, M.J. Wierman, The information entropy, rough entropy and knowledge granulation in incomplete information system, International Journal of General Systems 35 (6) (2006) 641-654.

[8] T.Y. Lin, Introduction to special issues on data mining and granular computing, International Journal of Approximate Reasoning 40 (2005) 1-2.

[9] Z. Pawlak, Rough sets: Theoretical Aspects of Reasoning about Data, London, Kluwer Academic Publisher, 1991.

[10] Y.H. Qian, J.Y. Liang, C.Y. Dang, Knowledge structure, knowledge granulation and knowledge distance in a knowledge base, International Journal of Approximate Reasoning 50(1) (2009) 174-188.

[11] Y.H. Qian, J.Y. Liang, W. Pedrycz, C.Y. Dang, Positive approximation: an accelerator for attribute reduction in rough set theory. Artificial Intelligence 174 (2010) 597-618.

[12] Y.H. Qian, J.Y. Liang, C.Y. Dang, Consistency measure, inclusion degree and fuzzy measure in decision tables. Fuzzy Sets and Systems 40(2) (2008) 2353-2377.

[13] Y. Y. Yao, Information granulation and rough set approximation, International Journal of Intelligent Systems 16 (2001) 87-104.