

## Apply inversion order number Genetic Algorithm to the Job Shop Scheduling Problem

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**Abstract**—Through analyzing the present genetic operators to solve the Job Shop Scheduling Problem, an inversion order number Genetic Algorithm is proposed. In view of the quality of the inversion order number, this algorithm measures the population diversity by the relative inversion order number. It uses the information provided by the inversion order number of the individual and the offspring is generated. This algorithm not only satisfies the characteristic of the Job Shop Scheduling Problem, but also develops the search capacity of Genetic Algorithm. The computation results validate the effectiveness of the proposed algorithm.

**Keywords**—Genetic Algorithm;the Job Shop Scheduling Problem;inversion order number

### I. INTRODUCTION

As an important research subject in the fields of Computer Integrated Manufacturing System, the Job Shop Scheduling Problem is the most commonly encountered scheduling problems. Because it is helpful to develop machine's utility and the ability to manage the factories, the Job Shop Scheduling Problem receives researchers' increasing attention. However, this problem has several important characteristics. It not only need satisfy some constraints, but also has dynamic production environments and complex solutions, which increase the problem's solving difficulty.

Since David applied Genetic Algorithm to the Job Shop Scheduling Problem successfully in 1983 firstly, Genetic Algorithm has been an effective scheduling method<sup>[1]</sup>. However, to the simple genetic algorithm, the rate and quality of solution for the problem is constrained by reason of the early convergence. Many researchers focused their attention on finding a solution to the problem by improving the capability of Genetic Algorithm<sup>[2,3]</sup>. As we known, genetic scheduling operators adopt the similar process with genetic operators solving to TSP because the Job Shop Scheduling Problem and TSP belong to the class of sequence problem. Because each of them has different objective criterion and constraints, it needs to design a new algorithm to solve the Job Shop Scheduling Problem efficiently considering its specific

characteristics. According to this, an inversion order number Genetic Algorithm is proposed. This algorithm is derived from the theory of inversion order number. In view of comparing the different inverse order sequence among the individuals, the better individual's structure is tailored to generate corresponding offspring. As a result, this genetic operator is expected to improve the search efficacy of genetic scheduling algorithm for considering the latent information..

### II. THE JOB SHOP SCHEDULING PROBLEM

The  $n \times m$  Job Shop Scheduling Problem (or JSSP) can be described by a set of  $n$  jobs or items that are to be processed on a set of  $m$  machines. Each job will have a set of constraints on the order in which machines can be used and a given processing time on each machine. Its aim is to find the sequence of jobs on each machine in order to optimize the objective function<sup>[4]</sup>.

The constraints of Job Shop Scheduling Problem are as follows.

- Each machine can process only one job at a time.
- Each job must be processed on a machine at fixed time.
- The processing course is an uninterrupted duration.

### III. INVERSION ORDER NUMBER GENETIC ALGORITHM

#### A. Relevant Research

Since the benchmark of Job Shop Scheduling Problem were presented by Fisher and Thomson in 1963, many algorithm have been used to solve this problem, such as local hill-climbing, simulated annealing, neural network, and so on. But these approaches may obtain bad search efficiency and seem to produce the phenomenon of combination explosive.

Genetic Algorithm was proposed by J. Holland, which simulated the genetic process of biological organisms<sup>[5]</sup>. As a powerful heuristic search approach, it can obtain good quality quickly and better than other

algorithm. So Genetic algorithm have been applied to guide search process successfully in developing heuristic for the Job Shop Scheduling Problem. However, Genetic Algorithm solving Job Shop Scheduling Problem suffered from convergence to local optimum. Besides that, the difficulty in this subject arises from the question of how to represent Job Shop Scheduling Problem and how to design genetic operators. Many chromosome representation schemes have been proposed for the Job Shop Scheduling Problem. Nakano and Yamada solved it by a conventional Genetic Algorithm using binary representation. This algorithm applied conventional genetic operators, such as 1-point, 2-point and uniform crossovers without any modification. The operators worked easily, but it could generate the illegal schedules. Lately, the Job Shop Scheduling Problem is viewed as an ordering problem just like the Traveling Salesman Problem. A schedule can be represented by Operation-based representation, Job-based representation, machine-based representation, etc. Thus genetic operators are similar to solving TSP problem such as ordered crossover, partially matched crossover. But there are different fitness function and encode method between the two problems, such as the gene sequence in TSP denotes the routes rank and the latter denotes the job's (machine's/operator's) prior scheduling order to different coding methods. So specific crossover should be adapted to specific problem in which can improve performance of the Genetic Algorithm. It means that the special operators should be designed to the Job Shop Scheduling Problem.

It is known that Job Shop Scheduling Problem is one of the NP problems and is included in a variety of complex manufacturing scheduling domain<sup>[6]</sup>. Each product has a certain number of parts that must be processed through a set of given machines. To get optimal solutions rapidly, more complex constraints and dynamic state should be taken into account. Due to No Free Lunch, the characteristic of specific problem should be considered within the searching process. To solve the Job Shop Scheduling Problem more efficiently, we should consider following problems in the process of Genetic Algorithm.

- How to get appropriate represent strategy by which can keep away from generating illegal offspring?
- Which kind of information may be availed to develop the ability of producing improved solutions?
- How to design genetic operators according to the scheduling problem's characteristic?

### B. Description of Inversion Order Number Genetic Algorithm

In above section, we analyzed previous genetic scheduling algorithm. There we introduce an inversion order number Genetic Algorithm matching with its encoding representation. This algorithm evaluates the variance between individuals with their difference inverse number sequence. It deals with the random choice of crossover (mutation) points based on gene regular sequence.

#### 1) Preliminary knowledge

Following notation are used to formulate the inversion order number genetic algorithm mathematically<sup>[6]</sup>.

**Definition 3.1.** Let a set of vector  $\{P_1, P_2, \dots, P_n\}$  denote the sequence  $\{1, 2, \dots, n\}$ . If  $i > j, P_i > P_j$  is satisfied. It denotes there is an inverse. The sum of inverses is named as inversion order number.

**Theorem 3.1.** If interchange two elements to a set of vector randomly, a new set of vector can be found. This method named Interchange. Each Interchange can change the odd/even of inversion order number.

These notations depict the formula of inverse order number to a set of vector. It inspires a instructional way for expressing the variance between genes located on different point. The theory of inversion order number is rarely introduced to some practical fields<sup>[7]</sup>. In 1996, J.S. Liu applied the inverse order number to measure the consistency in group orderings<sup>[8]</sup>.

**Definition 3.2.** Assume there are  $m$  individuals and  $n$  chosen solutions. Let  $G$  denotes the sequence set. Let  $E_i, E_j$  denote the  $i$ th and the  $j$ th chosen

solution ( $1 \leq i, j \leq n$ ). Set  $Q_i \triangleq \sum_{k=1}^m q_{ik}$  which is referred as the relative inverse order number  $E_i$ . To the set  $G = (E_1, E_2, \dots, E_m)$ , if  $Q_i < Q_j$ , it shows that  $E_i$  is prior to  $E_j$ . Otherwise, it shows that  $E_j$  is prior to  $E_i$ .

**Definition 3.3.** If  $Q = O$ , the sequence of  $m$  individuals  $E_1, E_2, \dots, E_m$  have absolute consistency. Where

$$Q \triangleq \sum_{i=1}^{m-1} \sum_{j=i+1}^m q_{ij} .$$

**Theorem 3.2.** Let  $\delta = \frac{4Q}{nm(n-1)(m-1)}$ . To the given

$G$ , If  $\delta < \frac{1}{2}$ , then the population has the relative consistency. Otherwise, it hasn't.

2) *Inversion order number genetic algorithm*

For an optimal schedule is always active, the search space can be safely limited to the set of all active schedules. So we propose a modified Operation-based representation. Firstly, it encodes a schedule as a sequence of operations. Then, through the relation between the operation with its location, the chromosomes are consist of no-repeated constants. This encoding method shows that another active schedule can be found while the gene's order is exchanged. Accordingly, it is essential to change the gene's sequence in genetic evolution process to a certain extent. For example, to  $2 \times 3$  Job Shop Scheduling Problem, let  $(1,1,2,2,3,3)$  be all the operations, let  $(1,2,3,4,5,6)$  be its genes locations, let  $O_{i,j}$  be the  $j$ th operation. The relation between the operation with its location is as TABLE I.

TABLE I. THE OPERATION WITH ITS LOCATION

operation	1	1	2	2	3	3
location	1	2	3	4	5	6
$O_{i,j}$	1,1	1,2	2,1	2,2	3,1	3,2

To the chromosome  $(3,1,6,4,2,5)$ , its relative operation set is  $(2,1,3,2,1,3)$ .

After finished initial solutions, it is indispensable to design genetic operator. There we introduced inversion order number genetic operators considering that the genes sequence to any chromosome denotes the priority scheduling sequence.

a) *The depulication operator*

The duplication operator uses the roulette wheel selection method. In the duplication course, comparing  $\delta$  with  $\frac{1}{2}$  is the measure criterion to satisfy the population diversity. The duplication process is as follows:

- Calculate all the individuals' inversion order number and sort them.
- Calculate  $\delta$  to Definition 3.3.
- If  $\delta \geq \frac{1}{2}$ , after adjusting the mutation probability, the offsprings can be found by the roulette wheel selection method based on their relative inversion order numbers. Otherwise,

the offsprings can be found the roulette wheel selection method based on their fitness.

b) *The crossover operator*

The crossover operator is described as follow.

- Select crossover parents for crossover with the crossover probability. According to the sequence of the individual's relative inversion order number, choice each two individuals as the two needed parent chromosomes. Let the parent chromosomes be

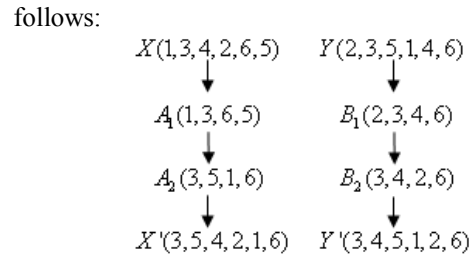
$$X(x_1, x_2, \dots, x_j), Y(y_1, y_2, \dots, y_l).$$

- Select two crossover points  $r_1, r_2$  randomly.
- If  $\delta \geq \frac{1}{2}$ , let the parents' genes between  $r_1, r_2$  be the offsprings' genes. Get crossover subset  $A_1(x_1, x_2, \dots, x_{r_1}, \dots, x_{r_2}, \dots, x_l)$  from the parent's genes besides  $r_1, r_2$ . Choose the same genes as  $A_1$  from  $Y$  to obtain the subset  $A_2$ . Let  $A_2$  be the rest genes of the offspring  $X'(x'_1, x'_2, \dots, x'_i, \dots, x'_j)$ . By the same way, the offspring  $Y'(y'_1, y'_2, \dots, y'_i, \dots, y'_l)$  is found.
- Otherwise, exchange the genes between  $r_1, r_2$  to get the two offsprings.

For example, assume

$$X(1,3,4,2,6,5), Y(2,3,5,1,4,6), r_1=3, r_2=5.$$

If  $\delta \geq \frac{1}{2}$ , then the offspring  $X', Y'$  can be got as follows:



The advantage of this crossover operator is:

- It voids the loss of good quality in several individuals. Because the crossover parent are evaluated the individuals' inversion order number, better individuals may usually be kept to crossover.
- It enhances search ability and reduces computation time. Based on the method, the diversity of every population is strengthened. It voids the algorithm to local optimum and illegal solution.

c) *The mutation operator*

The aim of mutation operator is mainly to improve the diversity of current population. There we adopts inversion order number mutation operator.

The mutation parents are selected with mutation probability which is adjusted to population's average inversion order number. Assume the parent be  $Z(z_1, z_2, \dots, z_l)$  and measure its inverse sequence to the current optimal solution. By choosing two mutation points randomly, reverse the corresponding genes' order. So the mutation offspring  $Z'$  is found. It is beneficial to take into account inversion order number after computing the mutation parent. Because the mutation operator decides the better segment through evaluating their inversion value, this method not only manipulates easily, but also changes the operations' order clearly.

IV. THE EXPERIMENTAL RESULTS

To validate the efficiency of inverse number Genetic Algorithm, we consider the Job Shop Scheduling Problem in [9] firstly. The experiment were performed using the following configuration: Popsiz=20, Pc=0.6, Pm=0.1, stopping criteria is maxgen=300, each program run 10 times. The result can get the optimum solution 98s better than [9]. Its chromosome is depicted as Figure 1.

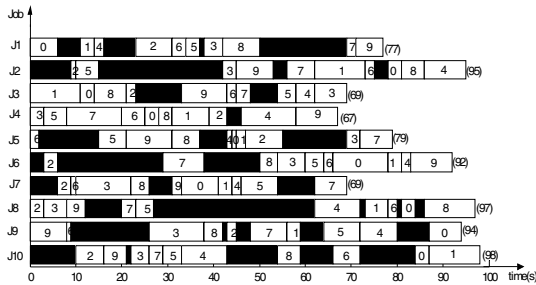


Figure 1. the best scheduling solution

Compare with conventional Genetic Algorithm, the result is depicted as TABLE II.

TABLE II. THE EXPERIMENT RESULT WITH SGA AND INVERSE NGA

Algorithm	Best fitness	Average fitness	Worst fitness	Convergence rate	Average Convergence generation
SGA	102	103.5	105	40	57.5
Inverse NGA	98	103	105	100	37.0

The better fitness can be found with inversion order number Genetic Algorithm than the algorithm in [9].

Secondly, to Ft06, La05 and La14, we use the following parameter:

Popsiz=40, Pc=0.9, Pm=0.1, maxgen=40.

The experiment results are as TABLE III.

TABLE III. THE EXPERIMENT RESULTS OF FOUR BENCHMARKS

Problem	Algorithm	Best	Average	worse	Numbers to find best value
Ft06	Refs [10]	55	56.5	59	3
	ION-GA	55	55.8	58	4
La05	Refs [10]	593	594.45	609	17
	ION-GA	593	593.15	596	19
La10	Refs [10]	958	958.6	962	8
	ION-GA	958	958.3	960	8
La14	Refs [10]	1292	1292.4	1296	9
	ION-GA	1292	1292	1292	10

From TABLE II and TABLE III, they showed our result increases the search speed and gets optimum solution rapidly than the algorithms in [9] and [10].

The inversion order number Genetic Algorithm can measure the varieties of individuals and use the specific good genotype properly based on the inverse sequence. While analyzing the representation of scheduling problem, the new genetic operators are in favor of assuring the diversity of the population. It leads us to search the optimum solution effectively. Different from conventional Genetic Algorithm to solving scheduling problems, it is a good idea to introduce the same operation toward the quality of Job Shop Scheduling Problem. So this algorithm can be a heuristic strategy to solve practical manufacturing problem.

V. CONCLUSION

Through describing the Job Shop Scheduling Problem, the inversion order number Genetic Scheduling Algorithm is presented according to the characteristic of encoding representation. This algorithm explores the variation between the individuals through measuring the inverse number of each individual. It is a promising technology to solve different problems using specific method since they have the similar characteristics.

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