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# Inclusion degree: a perspective on measures for rough set data analysis

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## Abstract

Rough set data analysis is one of the main application techniques arising from rough set theory. In this paper we introduce a concept of inclusion degree into rough set theory and establish several important relationships between the inclusion degree and measures on rough set data analysis. It is shown that the measures on rough set data analysis can be reduced to the inclusion degree. © 2002 Published by Elsevier Science Inc.

*Keywords:* Rough sets; Inclusion degree; Data analysis; Measure

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## 1. Introduction

Rough set theory, introduced by Pawlak (see [1,2]), is emerging as a powerful tool for reasoning about data. Rough set data analysis is one of the main application techniques arising from rough set theory (see [3–6]). It provides a technique for gaining insights into properties of data, dependencies, and significance of individual attributes in databases, and has important applications to artificial intelligence and cognitive sciences, as a tool for dealing with vagueness and uncertainty of facts, and in classification (see [2,7–11]). In order

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to analyze data effectively, many measures are defined in rough set data analysis, for example, accuracy measure of rough set, accuracy of approximation of classification, measure of dependency of attributes, measure of importance of attributes, and accuracy and coverage of decision rule, etc. Although these measures can be applied to justifying effectiveness of data analysis, it is unclear what is the main foundation behind these measures and whether they have any common characteristics.

Answers to these questions will be very helpful for people to understand the essence of rough set data analysis and to employ rough set data analysis to solve practical problems effectively. In this paper, a concept of inclusion degree is introduced into rough set data analysis and several important relationships between the inclusion degree and measures on rough set data analysis are established. It is shown that the measures on rough set data analysis can be reduced to the inclusion degree.

## 2. Inclusion degree

An approximate mereological calculus called rough mereology (i.e., theory of rough inclusions) has been proposed as a formal treatment of the hierarchy of relations of being a part in a degree (see [12–14]). The degree of inclusion is a particular case of inclusion in a degree (rough inclusion) basic for rough mereology. The concept of inclusion degree based on partially ordered relation was proposed in [15] for approximate reasoning. By a slight adjustment of this concept, we introduce a definition of inclusion degree into rough set data analysis.

A partial order on a set  $L$  is a binary relation  $\preceq$  with the following properties:

- $x \preceq x$  (reflexive),
- $x \preceq y$  and  $y \preceq x$  imply  $x = y$  (antisymmetric), and
- $x \preceq y$  and  $y \preceq z$  imply  $x \preceq z$  (transitive).

**Definition 1.** Let  $(L, \preceq)$  be a partially ordered set. If, for any  $a, b \in L$ , there is a real number  $D(b/a)$  with the following properties:

- (1)  $0 \leq D(b/a) \leq 1$ ,
  - (2)  $a \preceq b$  implies  $D(b/a) = 1$ ,
  - (3)  $a \preceq b \preceq c$  implies  $D(a/c) \leq D(a/b)$ , and
  - (4)  $a \preceq b$  implies  $D(a/c) \leq D(b/c)$  for  $\forall c \in L$ ,
- then  $D$  is called an inclusion degree on  $L$ .

In Definition 1, (1) is normalization for inclusion degree; (2) states the property of consistency between inclusion degree and standard inclusion; and (3) and (4) state the property of monotonicity of inclusion degree.

Inclusion degree is practically a measure on partially ordered relation, but it has more important applications than partially ordered relation.

**Example 1.** Let  $U$  be a finite set,  $F = \{X \mid X \subseteq U\}$ , and  $\subseteq$  is a partially ordered relation on  $F$ . For  $\forall X, Y \in F$ , we define

$$D_0(Y/X) = \begin{cases} \frac{|Y \cap X|}{|X|} & \text{if } X \neq \emptyset, \\ 1 & \text{if } X = \emptyset, \end{cases} \tag{1}$$

where  $|X|$  denotes the cardinality of  $X$ .

It is easy to see that  $D_0$  is an inclusion degree on  $F$ . In [12],  $D_0$  is regarded as a particular case of rough inclusions.

Rough inclusions and inclusion degree have some common characteristics on measure, but rough inclusions is more appropriate for reasoning about complex structures, inclusion degree is more appropriate for measure on partially ordered relations.

### 3. Basic concepts of rough sets

Formally, an information system is an ordered quadruple  $S = (U, A, V, f)$ , where:

$U$  is a non-empty finite set of objects;

$A$  is a non-empty finite set of attributes;

$V$  is the union of attribute domains, i.e.,  $V = \bigcup V_a$  for every  $a \in A$ , where  $V_a$  denotes the domain of the attribute  $a$ ;

$f : U \times A \rightarrow V$  is an information function which associates an unique value of each attribute with every object belonging to  $U$ , i.e.,  $\forall a \in A$  and  $x \in U$ ,  $f(x, a) \in V_a$ .

Each subset of attributes  $P \subseteq A$  determines a binary indiscernibility relation  $IND(P)$  as follows:

$$IND(P) = \{(x, y) \in U \times U \mid \forall a \in P, f(x, a) = f(y, a)\}.$$

Obviously  $IND(P)$  is an equivalence relation on the set  $U$  and

$$IND(P) = \bigcap_{a \in P} IND(\{a\}).$$

The relation  $IND(P)$ ,  $P \subseteq A$ , constitutes a partition of  $U$ , which we will denote by  $U/IND(P)$ . Any element from  $U/IND(P)$  will be called an equivalence class. Let  $[x]_{IND(P)}$  denote the equivalence class of the relation  $IND(P)$  containing the element  $x$ .

Let  $P \subseteq A$  and  $X \subseteq U$ . Then  $P$ -lower and  $P$ -upper approximations of  $X$  are defined respectively as follows:

$$\underline{P}X = \bigcup \{Y \mid Y \in U/\text{IND}(P), Y \subseteq X\}$$

and

$$\overline{P}X = \bigcup \{Y \mid Y \in U/\text{IND}(P), Y \cap X \neq \emptyset\}.$$

The set  $BN_P(X) = \overline{P}X - \underline{P}X$  will be called the  $P$ -boundary of  $X$ . The set  $\underline{P}X$  is the set of all elements of  $U$ , which can be with classified certainty as elements of  $X$  with respect to the values of attributes from  $P$ ; and the set  $\overline{P}X$  consists of those elements of  $U$  which can be possibly defined as elements of  $X$  with respect to the values of attributes from  $P$ . Finally,  $BN_P(X)$  is the set of elements which can be classified neither in  $X$  nor in  $U - X$  on the basis of the values of attributes from  $P$ .

#### 4. Relationships between inclusion degree and measures on rough set data analysis

4.1. Accuracy measure of rough set and degree of rough belonging can be reduced to inclusion degree

Let  $S = (U, A, V, f)$  be an information system,  $P \subseteq A$ , and  $X \subseteq U$ . The accuracy measure of rough set  $X$  with respect to  $P$  is defined as

$$\alpha_P(X) = \frac{|\underline{P}X|}{|\overline{P}X|}, \quad (2)$$

where  $X \neq \emptyset$ .

It is easy to show that

$$\alpha_P(X) = \frac{|\underline{P}X \cap \overline{P}X|}{|\overline{P}X|} = D_0(\underline{P}X/\overline{P}X).$$

The degree of rough belonging of  $x \in X$  about  $X$  with respect to  $P$  is defined as

$$\mu_X^P(x) = \frac{|X \cap [x]_{\text{IND}(P)}|}{|[x]_{\text{IND}(P)}|}. \quad (3)$$

It follows obviously that

$$\mu_X^P(x) = D_0(X/[x]_{\text{IND}(P)}).$$

Hence,  $\alpha_P(X)$  and  $\mu_X^P(x)$  can be reduced to inclusion degree.

4.2. Accuracy of approximation of classification and quality of approximation of classification can be reduced to inclusion degree

Let  $S = (U, A, V, f)$  be an information system, and  $P \subseteq A$ . Let  $Y = \{Y_1, Y_2, \dots, Y_n\}$  be a classification, or partition, of  $U$ . The origin of this classification is independent from attributes contained in  $P$ . Subsets  $Y_i, i = 1, 2, \dots, n$ , are classes of classification  $Y$ . By  $P$ -lower and  $P$ -upper approximations of  $Y$  in  $S$  we mean sets  $\underline{P}Y = \{\underline{P}Y_1, \underline{P}Y_2, \dots, \underline{P}Y_n\}$  and  $\overline{P}Y = \{\overline{P}Y_1, \overline{P}Y_2, \dots, \overline{P}Y_n\}$ , respectively. The coefficient

$$d_P(Y) = \frac{\sum_{i=1}^n |\underline{P}Y_i|}{\sum_{i=1}^n |\overline{P}Y_i|} \tag{4}$$

is called the accuracy of approximation of classification  $Y$  by the set of attributes  $P$ , or in short, accuracy of classification. It expresses the possible correct decisions when the classified objects possess the set of attributes  $P$ .

The coefficient

$$\gamma_P(Y) = \frac{\sum_{i=1}^n |\underline{P}Y_i|}{|U|} \tag{5}$$

is called the quality of approximation of classification  $Y$  by the set of attributes  $P$ , or in short, quality of classification. It expresses the percentage of objects which can be correctly classified into class  $Y$  employing the set of attributes  $P$ .

Let  $Y = \{Y_1, Y_2, \dots, Y_n\}$  be a classification, or partition, of  $U$ . Let  $F = \{\{F_1, F_2, \dots, F_n\} | F_i \subseteq Y_i, i = 1, 2, \dots, n\}$ ,  $X = \{X_1, X_2, \dots, X_n\} \in F$  and  $Z = \{Z_1, Z_2, \dots, Z_n\} \in F$ .

A partially ordered relation  $\preceq$  on  $F$  is defined as follows:

$$X \preceq Z \text{ if and only if } X_i \subseteq Z_i, \quad i = 1, 2, \dots, n.$$

For  $\forall X, Z \in F$ , define

$$D_1(X/Z) = \frac{|(\bigcup_{i=1}^n X_i) \cap (\bigcup_{i=1}^n Z_i)|}{|\bigcup_{i=1}^n Z_i|}. \tag{6}$$

It can be easily shown that  $D_1$  is inclusion degree on  $F$ .

Since  $d_P(Y) = D_1(\underline{P}Y/\overline{P}Y)$  and  $\gamma_P(Y) = D_1(\underline{P}Y/Y)$ ,  $d_P(Y)$  and  $\gamma_P(Y)$  can be reduced to inclusion degree.

4.3. Measure of dependency of attributes and measure of importance of attributes can be reduced to inclusion degree

An information system  $S = (U, A, V, f)$  can be seen as a decision table assuming that  $A = C \cup D$  and  $C \cap D = \emptyset$ , where  $C$  is called the set of condition

attributes, and  $D$  is called the set of decision attributes. Let  $P \subseteq C$  and  $Q \subseteq D$ . The measure of dependency between  $P$  and  $Q$  is defined as

$$\gamma(P, Q) = \frac{|POS_P(Q)|}{|U|}, \quad (7)$$

where  $POS_P(Q) = \bigcup \{PY | Y \in U/IND(Q)\}$ .

Let  $F$  denote the set of all partitions on  $U$ ,  $X = \{X_1, X_2, \dots, X_n\} \in F$  and  $Z = \{Z_1, Z_2, \dots, Z_m\} \in F$ . A partially ordered relation  $\preceq$  on  $F$  is defined as follows:

$X \preceq Z$  if and only if, for  $\forall X_i \in X$ , there exists  $Z_j \in Z$  such that  $X_i \subseteq Z_j$ .

For  $\forall X, Z \in F$ , define

$$D_2(Z/X) = \frac{\left| \bigcup_{Z_j \in Z} \left( \bigcup_{X_i \subseteq Z_j} X_i \right) \right|}{|U|}. \quad (8)$$

We prove in the following that  $D_2$  is inclusion degree on  $F$ .

(1) Obviously,  $0 \leq D_2(Z/X) \leq 1$ .

(2) Let  $X = \{X_1, X_2, \dots, X_n\} \in F$ ,  $Z = \{Z_1, Z_2, \dots, Z_m\} \in F$  and  $X \preceq Z$ . Then we have  $m \leq n$  and there exists a partition  $E = \{E_1, E_2, \dots, E_m\}$  of  $\{1, 2, \dots, n\}$  such that

$$Z_j = \bigcup_{i \in E_j} X_i, \quad j = 1, 2, \dots, m.$$

Hence

$$\bigcup_{Z_j \in Z} \left( \bigcup_{X_i \subseteq Z_j} X_i \right) = \bigcup_{Z_j \in Z} Z_j = U.$$

Thus

$$D_2(Z/X) = \frac{|U|}{|U|} = 1.$$

(3) Let  $X = \{X_1, X_2, \dots, X_n\} \in F$ ,  $Z = \{Z_1, Z_2, \dots, Z_m\} \in F$ ,  $Y = \{Y_1, Y_2, \dots, Y_l\} \in F$  and  $X \preceq Z \preceq Y$ . Then we have  $l \leq m$  and there exists a partition  $E = \{E_1, E_2, \dots, E_l\}$  of  $\{1, 2, \dots, m\}$  such that

$$Y_j = \bigcup_{i \in E_j} Z_i, \quad j = 1, 2, \dots, l.$$

We show in the following that

$$\bigcup_{X_j \in X} \left( \bigcup_{Y_i \subseteq X_j} Y_i \right) \subseteq \bigcup_{X_j \in X} \left( \bigcup_{Z_i \subseteq X_j} Z_i \right). \quad (9)$$

Let  $X_j \in X, Y_{i_0} \in Y$  and  $Y_{i_0} \subseteq X_j$ . From  $Z \preceq Y$ , it follows that  $Y_{i_0} = \bigcup_{i \in E_{i_0}} Z_i$ . For  $\forall i_1 \in E_{i_0}$ , we have  $Z_{i_1} \subseteq Y_{i_0}$  and  $Z_{i_1} \subseteq X_j$ , hence,

$$Z_{i_1} \subseteq \bigcup_{X_j \in X} \left( \bigcup_{Z_i \subseteq X_j} Z_i \right),$$

i.e.,

$$Y_{i_0} \subseteq \bigcup_{X_j \in X} \left( \bigcup_{Z_i \subseteq X_j} Z_i \right).$$

This completes the proof of (9).

From (9), we have

$$D_2(X/Y) \leq D_2(X/Z).$$

(4) Let  $X = \{X_1, X_2, \dots, X_n\} \in F, Z = \{Z_1, Z_2, \dots, Z_m\} \in F$  and  $X \preceq Z$ . For  $\forall Y = \{Y_1, Y_2, \dots, Y_l\} \in F$ , we have

$$\bigcup_{X_j \in X} \left( \bigcup_{Y_i \subseteq X_j} Y_i \right) \subseteq \bigcup_{Z_j \in Z} \left( \bigcup_{Y_i \subseteq Z_j} Y_i \right). \tag{10}$$

In fact, let  $X_j \in X, Y_{i_0} \in Y$  and  $Y_{i_0} \subseteq X_j$ . From  $X \preceq Z$ , it follows that there exists  $Z_{j_0} \in Z$  such that  $X_j \subseteq Z_{j_0}$ . Hence,  $Y_{i_0} \subseteq Z_{j_0}$ , i.e.,

$$Y_{i_0} \subseteq \bigcup_{Z_j \in Z} \left( \bigcup_{Y_i \subseteq Z_j} Y_i \right).$$

This implies (10).

From (10), we have

$$D_2(X/Y) \leq D_2(Z/Y).$$

Hence,  $D_2$  is inclusion degree on  $F$ .

Since  $\gamma(P, Q) = D_2((U/\text{IND}(Q))/(U/\text{IND}(P)))$ ,  $\gamma(P, Q)$  can be reduced to inclusion degree, i.e., degree of partition  $U/\text{IND}(Q)$  includes partition  $U/\text{IND}(P)$ .

**Remark.** Let  $P \rightarrow Q$  denote functional dependency between  $P$  and  $Q$ . Then  $P \rightarrow Q$  if and only if  $D_2((U/\text{IND}(Q))/(U/\text{IND}(P))) = 1$ .

In rough set data analysis, the measure of importance of condition attributes  $C' \subseteq C$  with respect to decision attributes  $D$  is defined as follows:

$$\gamma(C, D) - \gamma(C - C', D). \tag{11}$$

In particular, when  $C' = \{c\}$ ,  $\gamma(C, D) - \gamma(C - \{c\}, D)$  is the measure of importance of attribute  $c \subseteq C$  with respect to  $D$ .

Since

$$\begin{aligned} \gamma(C, D) - \gamma(C - C', D) &= D_2((U/\text{IND}(D))/(U/\text{IND}(C))) \\ &\quad - D_2((U/\text{IND}(D))/(U/\text{IND}(C - C'))), \end{aligned}$$

$\gamma(C, D) - \gamma(C - C', D)$  can be reduced to computation of inclusion degree.

*4.4. Measure of the relative degree of misclassification can be reduced to inclusion degree*

Let  $X$  and  $Y$  be non-empty subsets of a finite universe  $U$ . The measure  $c(X, Y)$  of the relative degree of misclassification of the set  $X$  with respect to set  $Y$  (see [16]) defined as

$$c(X, Y) = \begin{cases} 1 - \frac{|Y \cap X|}{|X|} & \text{if } |X| > 0, \\ 0 & \text{if } |X| = 0. \end{cases} \tag{12}$$

It can be easily shown that

$$c(X, Y) = 1 - D_0(Y/X) = D_0((U - Y)/X).$$

This means that  $c(X, Y)$  can be reduced to inclusion degree.

**Remark.** Let  $0 \leq \beta < 0.5$ . Then  $c(X, Y) \leq \beta$  if and only if  $D_0(Y/X) \geq 1 - \beta$ . Thus, the variable precision rough set model (see [16]) can be expressed by inclusion degree as follows.

Let  $X \subseteq U$  and  $R$  be an equivalence relation on  $U$ . The  $\beta$ -lower approximation of the set  $X$  is defined as

$$\underline{R}_\beta X = \bigcup \{E \in U/\text{IND}(R) \mid D_0(X/E) \geq 1 - \beta\},$$

and the  $\beta$ -upper approximation of the set  $X$  is defined as

$$\overline{R}_\beta X = \bigcup \{E \in U/\text{IND}(R) \mid D_0(X/E) > \beta\}.$$

Consequently, the  $\beta$ -boundary region of  $X$  is given by

$$\text{BNR}_\beta X = \bigcup \{E \in U/\text{IND}(R) \mid \beta < D_0(X/E) < 1 - \beta\}.$$

The  $\beta$ -negative region of  $X$  is defined as a complement of the  $\beta$ -upper approximation, i.e.,

$$\text{NEGR}_\beta X = \bigcup \{E \in U/\text{IND}(R) \mid D_0(X/E) \leq \beta\}.$$

*4.5. Accuracy and coverage of decision rule can be reduced to inclusion degree*

Let  $S = (U, A, V, f)$  be a decision table with  $A = C \cup D$  and  $C \cap D = \emptyset$ , where  $C$  is the set of condition attributes and  $D$  is the set of decision attributes.



Let  $U/\text{IND}(C) = \{X_1, X_2, \dots, X_n\}$  and  $U/\text{IND}(D) = \{Y_1, Y_2, \dots, Y_m\}$  denote the partitions on  $U$  induced respectively by the equivalence relations  $\text{IND}(C)$  and  $\text{IND}(D)$ . Expression  $\text{Des}_C(X_i) \rightarrow \text{Des}_D(Y_j)$  is called the  $(C, D)$ -decision rule in  $S$ , where  $\text{Des}_C(X_i)$  and  $\text{Des}_D(Y_j)$  are unique descriptions of the classes  $X_i$  and  $Y_j$ , respectively ( $i = 1, 2, \dots, n; j = 1, 2, \dots, m$ ). The set of decision rules  $\{r_{ij}\}$  for each class  $Y_j$  ( $j = 1, 2, \dots, m$ ) can be defined as

$$\{r_{ij}\} = \{\text{Des}_C(X_i) \rightarrow \text{Des}_D(Y_j) \mid Y_j \cap X_i \neq \emptyset, i = 1, 2, \dots, n\}.$$

A decision rule  $r_{ij}$  is deterministic iff  $Y_j \cap X_i = X_i$ , and  $r_{ij}$  is non-deterministic otherwise.

The accuracy and coverage of decision rule  $r_{ij}$  (see [17]) are defined respectively as

$$\alpha_{X_i}(Y_j) = \frac{|Y_j \cap X_i|}{|X_i|}, \quad \kappa_{X_i}(Y_j) = \frac{|Y_j \cap X_i|}{|Y_j|}. \quad (13)$$

It is notable that  $\alpha_{X_i}(Y_j)$  measures the degree of sufficiency of a proposition,  $\text{Des}_C(X_i) \rightarrow \text{Des}_D(Y_j)$ , and that  $\kappa_{X_i}(Y_j)$  measures the degree of its necessity. It can be easily shown that

$$\alpha_{X_i}(Y_j) = D_0(Y_j/X_i), \quad \kappa_{X_i}(Y_j) = D_0(X_i/Y_j).$$

This means that  $\alpha_{X_i}(Y_j)$  and  $\kappa_{X_i}(Y_j)$  can be reduced to inclusion degree.

## 5. Conclusions

Rough set data analysis is one of the main application techniques arising from rough set theory. In this paper, the concept of inclusion degree has been introduced, several important relationships between inclusion degree and measures on rough set data analysis are established, and we have shown that the measures on rough set data analysis can be reduced to inclusion degree. These results will be very helpful for people to understand the essence of rough set data analysis, and can be regarded as the main foundation of measures which are defined for rough set data analysis. The introduction of inclusion degree will play a significant role in further research on rough set data analysis.

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